Справочный материал

1. Тригонометрические преобразования.

$\sin^2\alpha+\cos^2\alpha=1$; $\operatorname{tg}\alpha=\dfrac{\sin\alpha}{\cos\alpha}$; $1+\operatorname{tg}^2\alpha=\dfrac{1}{\cos^2\alpha}$; $\operatorname{ctg}\alpha=\dfrac{\cos\alpha}{\sin\alpha}$; $1+\operatorname{ctg}^2\alpha=\dfrac{1}{\sin^2\alpha}$;

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$; $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$; $1+\cos\alpha=2\cos^2\dfrac{\alpha}{2}$; $1-\cos\alpha=2\sin^2\dfrac{\alpha}{2}$;

$\operatorname{tg}\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{\sin\alpha}$; $\cos^2\alpha=\dfrac{1}{2}(1+\cos 2\alpha)$; $\sin^2\alpha=\dfrac{1}{2}(1-\cos 2\alpha)$; $\operatorname{tg}^2\alpha=\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}$; $\sin\alpha=\dfrac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}$; $\cos\alpha=\dfrac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}$;

$\operatorname{tg}\alpha=\dfrac{2\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}}$; $\operatorname{ctg}\alpha=\dfrac{1-\operatorname{tg}^2\frac{\alpha}{2}}{2\operatorname{tg}\frac{\alpha}{2}}$; $\sin\alpha\pm\sin\beta=2\sin\dfrac{\alpha\pm\beta}{2}\cos\dfrac{\alpha\mp\beta}{2}$; $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$;

$\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha-\beta}{2}\sin\dfrac{\alpha+\beta}{2}$; $\sin\alpha\pm\cos\beta=\sin\alpha\pm\sin\left(\dfrac{\pi}{2}+\beta\right)=2\sin\left(\dfrac{\alpha\pm\beta}{2}\pm\dfrac{\pi}{4}\right)\cos\left(\dfrac{\alpha\mp\beta}{2}\mp\dfrac{\pi}{4}\right)$;

$\sin\alpha\cos\beta=\dfrac{1}{2}\big(\sin(\alpha+\beta)+\sin(\alpha-\beta)\big)$; $\cos\alpha\cos\beta=\dfrac{1}{2}\big(\cos(\alpha+\beta)+\cos(\alpha-\beta)\big)$; $\sin\alpha\sin\beta=\dfrac{1}{2}\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big)$.

2. Формулы Маклорена

$(1+x)^\alpha=1+\alpha x+\ldots+\dfrac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}x^n+o(x^n)$; $\dfrac{1}{1-x}=1+x+\cdots+x^n+o(x^n)$;

$e^x=1+\dfrac{x}{1!}+\cdots+\dfrac{x^n}{n!}+o(x^n)$;

$\cos x=1-\dfrac{x^2}{2!}+\cdots+(-1)^n\dfrac{x^{2n}}{(2n)!}+o(x^{2n+1})$;

$\sin x=x-\dfrac{x^3}{3!}+\cdots+(-1)^n\dfrac{x^{2n+1}}{(2n+1)!}+o(x^{2n+2})$;

$\ln(1+x)=x-\dfrac{x^2}{2}+\cdots+(-1)^{n+1}\dfrac{x^n}{n}+o(x^n)$.