

1. Тригонометрические преобразования:

$$\sin^2 \alpha + \cos^2 \alpha = 1; \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}; 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}; \operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}; 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha};$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta;$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta;$$

$$1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}; 1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2};$$

$$\operatorname{tg} \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha};$$

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha); \sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha); \operatorname{tg}^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha};$$

$$\sin \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}; \cos \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}};$$

$$\operatorname{tg} \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 - \operatorname{tg}^2 \frac{\alpha}{2}}; \operatorname{ctg} \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{2 \operatorname{tg} \frac{\alpha}{2}};$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2};$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2};$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2};$$

$$\sin \alpha \pm \cos \beta = \sin \alpha \pm \sin \left(\frac{\pi}{2} + \beta \right) = 2 \sin \left(\frac{\alpha \pm \beta}{2} \pm \frac{\pi}{4} \right) \cos \left(\frac{\alpha \mp \beta}{2} \mp \frac{\pi}{4} \right);$$

$$\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta));$$

$$\cos \alpha \cos \beta = \frac{1}{2}(\cos(\alpha + \beta) + \cos(\alpha - \beta));$$

$$\sin \alpha \sin \beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta)).$$

2. Табличные интегралы:

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C; \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C; \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C;$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{arcsin} \frac{x}{a} + C;$$

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C;$$

$$\int \frac{dx}{\cos x} = \ln \left| \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + C.$$

3. Суммы:

$$1 + 2 + \dots + n = \frac{n(n+1)}{2};$$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}; q + q^2 + \dots + q^n = \frac{q^{n+1} - q}{q-1}.$$
