Threshold Functions and Operations in the Theory of Evidence *

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Abstract. The article introduces and discusses threshold belief and plausibility functions. When forming such functions, only focal elements that are "significant" for a given set are taken into account. The significance of focal elements is determined using a similarity measure and a threshold. Threshold functionals of uncertainty, external and internal conflicts, threshold rules of combination are introduced and considered on the basis of threshold functions of the theory of evidence. A number of examples are given to illustrate the use of threshold tools.

Keywords: Similarity measures · Theory of evidence.

1 Introduction

All focal elements that intersect with a given set are taken into account in the theory of evidence when assessing the plausibility that the true alternative belongs to the given set. However, some focal elements may have been formed inaccurately. If a focal element overlaps "weakly" (relative to some similarity measure) with a given set, then the degree of confidence that this focal element is important in assessing the plausibility of membership of a true alternative in a given set will be small. Therefore, the problem of taking into account the degree of intersection or inclusion of focal elements with a given set is relevant when forming the main functions of the theory of evidence (belief, plausibility, etc.).

This problem is related to the analysis of the sensitivity of the main functions of the theory of evidence to small changes in focal elements. Traditionally, sensitivity to small changes in focal elements is analyzed using generalization-specialization procedures [6]. But these procedures are performed on the body of evidence itself and do not take into account the degree of intersection or inclusion with a given set.

A similar problem of taking into account significant (i. e., having a large degree of intersection or inclusion) focal elements is relevant when performing operations of aggregating bodies of evidence, assessing conflict, degree of uncertainty, etc.

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The procedure for calculating plausibility functions taking into account only significant focal elements with respect to some similarity measure [2] will be discussed in this article. Similarly, focal elements that are "significantly" contained in a given set can be taken into account when forming a belief function. We will call such functions threshold, since they take into account focal elements for which the similarity measure exceeds a certain threshold. In general, threshold functions may not satisfy some important properties of evidence theory. The article discusses the conditions when these properties will be satisfied.

In addition, threshold functions give rise to the concepts of threshold functionals of uncertainty, external and internal conflicts of bodies of evidence, as well as threshold rules of combination. All these concepts are discussed in this article.

Taking into account the significant focal elements allows for a controlled reduction in the measure of uncertainty in the description of bodies of evidence compared to non-threshold functions.

2 Necessary Information from the Theory of Evidence

Let us recall the necessary information from the theory of evidence [10]. Let $X = \{x_1, ..., x_n\}$ be a finite set; 2^X be the set of all subsets on X; m: $2^X \to [0,1]$, $\sum_{A \in 2^X} m(A) = 1$, $m(\emptyset) = 0$ is a mass function; $\mathcal{A}$ is a set of all focal elements, i.e. $A \in \mathcal{A}$ if $m(A) > 0$; $F = (\mathcal{A}, m)$ is the body of evidence; $\mathcal{F}(X)$ be the set of all bodies of evidence on the X. The body of evidence $F = (\mathcal{A}, m)$ is symbolically convenient to represent in the form $F = \sum_{A \in \mathcal{A}} m(A) F_A$, where $F_A = (\{A\}, 1)$ is a categorical body of evidence.

The body of evidence $F = (\mathcal{A}, m)$ uniquely defines the belief function

$$Bel(A) = \sum_{B \subseteq A} m(B)$$

and its dual plausibility function

$$Pl(A) = 1 - Bel(\neg A) = \sum_{B \cap A \neq \emptyset} m(B). \tag{1}$$

It's always true that $Bel(A) \leq Pl(A) \ \ \forall A \subseteq X$, and the length of the interval $[Bel(A), Pl(A)]$ determines the degree of uncertainty of the event $x \in A$ [1].

3 Threshold Functions of Belief and Plausibility

The summation of the mass functions in the formula (1) is carried out over all focal elements that have a non-empty intersection with the given set. This sum may include focal elements that have small (relative to some measure) intersection compared to the measures of the intersecting sets themselves. Such elements can be considered insignificant for assessing the plausibility of belonging to a given set. If we want to form a plausibility function in which only significant elements are taken into account, then this can be done using the formula

$$Pl_h(A) = \sum_{B:s(A,B)>h} m(B), \tag{2}$$

where $h \in [0,1)$ and $s(A,B)$ is a measure (index) of similarity [2], satisfying the conditions: 1) $0 \le s(A,B) \le 1$; 2) $s(A,B) = 0 \Leftrightarrow A \cap B = \emptyset$; 3) $s(A,A) = 1$ $\forall A \ne \emptyset$ (or weaker condition $\max_B s(A,B) = s(A,A)$). The similarity measure must often (but not necessarily) satisfy the symmetry condition: $s(A,B) = s(B,A)$. Asymmetric similarity measures are called inclusion measures. Examples of similarity measures: a) Jaccard index $J(A,B) = \dfrac{|A \cap B|}{|A \cup B|}$; b) $s(A,B) = \dfrac{|A \cap B|}{|X|}$; c) $s(A,B) = \begin{cases} 1, A \cap B \ne \emptyset, \\ 0, A \cap B = \emptyset; \end{cases}$ d) Simpson coefficient $s(A,B) = \dfrac{|A \cap B|}{\min\{|A|, |B|\}}$; e) Sörensen inclusion measure $s(A,B) = \dfrac{|A \cap B|}{|B|}$; f) Sörensen coefficient $s(A,B) = \dfrac{2|A \cap B|}{|A| + |B|}$.

We will call a function of the form (2) the threshold plausibility function. The parameter $h \in [0,1)$ regulates the degree of similarity between the focal elements that are taken into account when calculating the plausibility and the given set. The larger h, the higher this degree of closeness.

Transformations (1), (2) can be represented in matrix form $\sum_B S(A,B)m(B) = \mathbf{Sm}$, where $\mathbf{S} = (S(A,B))_{A,B \in 2^X}$, $\mathbf{m}$ – $2^{|X|}$-dimensional column vector, the coordinates of which are the values of the mass function $m(A)$, $A \in 2^X$. For example, $Pl_h = \mathbf{S}_h\mathbf{m}$, where $\mathbf{S}_h = (S_h(A,B))_{A,B \in 2^X}$ and $S_h(A,B) = \begin{cases} 1, \text{ if } s(A,B) > h, \\ 0, \text{ otherwise.} \end{cases}$ Such matrix transformations were considered, for example, in [5].

Remark 1. In general, the transformation matrix can be non-binary. Its elements may depend on the measure of similarity between the focal elements and the set, the plausibility function of which is calculated. For example, $\widetilde{S_h}(A,B) = \begin{cases} s(A,B), & \text{if } s(A,B) > h, \\ 0, & \text{otherwise.} \end{cases}$ Note that the plausibility function $\widetilde{Pl_0} = \widetilde{\mathbf{S}_0}\mathbf{m}$ with subscript e) coincides with the pignistic probability [11] $\widetilde{Pl_0}(A) = \sum_B \frac{|A \cap B|}{|B|} m(B)$.

Properties of functions Pl_h:

1) $Pl_0 = Pl$. This follows from condition 2) for the similarity measure;
2) $Pl_{h_1}(A) \le Pl_{h_2}(A)$ $\forall A \in 2^X$ if $h_1 \ge h_2$.
3) $Pl_h(\emptyset) = 0$, but $Pl_h(X) \le 1$.

Property 3) means that the inequality $Pl_h(X) < 1$ will be true for some similarity measures s and sufficiently large $h \in [0,1)$. But, for example, equality $Pl_h(X) = 1$ $\forall h \in [0,1)$ is always true for the Simpson coefficient or index e).

4) if the similarity measure $s(A,B)$ is monotone with respect to A (i.e. $A' \subseteq A''$ implies that $s(A',B) \le s(A'',B)$), then the function Pl_h will be monotonic.

The monotonicity condition for the similarity measure $s(A,B)$ with respect to the first argument is certainly satisfied by measures b), c), e).

Let us find the threshold belief function Bel_h as dual to Pl_h. We have

$$Bel_h(A) = 1 - Pl_h(\neg A) = 1 - \sum_{B:s(\neg A,B)>h} m(B) = \sum_{B:s(\neg A,B)\le h} m(B). \quad (3)$$

In other words, the threshold belief function is formulated taking into account focal sets, which may contain a "small" number of elements not included in the considered set.

Remark 2. The classical concept of a belief function (including a threshold one) assumes that the summation in (3) must be performed over sets B that intersect A and satisfy the condition $s(\neg A, B) \le h$ in the threshold case. But the statement $s(\neg A,B) \le h \Rightarrow A \cap B \ne \emptyset$ is not true in the general case. It will be true, for example, for similarity measures c), d), e). These similarity measures are preferable to use in threshold belief functions.

Remark 3. Note that $Bel_h(X) = 1\ \forall h \in [0,1)$ is true for Bel_h and any similarity measure. But it may be $Bel_h(\emptyset) > 0$ for some similarity measures and some $h \in [0,1)$. This situation corresponds to the so-called open world concept, which is considered within the framework, for example, of the Transferable Belief Model [12] or the Generalized evidence theory [4]. At the same time, $Bel_0 = Bel$.

The duality relationship and property 2) imply that:

2') $Bel_{h_1}(A) \ge Bel_{h_2}(A)\ \forall A \in 2^X$ if $h_1 \ge h_2$;

4') if the similarity measure $s(A,B)$ is monotone with respect to A, then the function Bel_h will be monotone.

Remark 4. If the threshold belief and plausibility functions are defined on a finite set X, then they can be represented in matrix form. Let $0 = h_1 < \ldots < h_l = 1$ be an ordered set of different values of the similarity measure $s(A,B)$, $A, B \in 2^X$. It is clear that the values of $Bel_h(A)$ and $Pl_h(A)$ will not change within the intervals $h \in [h_j, h_{j+1})$, $j = 1, \ldots, l-1$. Let $\{A_i\}_{i=1}^{2^X-1}$ be the lexicographically ordered set of all proper subsets of X. Then the function Bel_h can be represented by the matrix $\mathbf{Bel} = (bel_{ij})$, where $bel_{ij} = Bel_{h_j}(A_i)$. The matrix $\mathbf{Pl}$ for Pl_h is formed in a similar way. Note that the partition $H = \{h_j\}_{j=0}^{l}$ and the size of the matrices depend only on the similarity measure s and $|X|$, but do not depend on evidence bodies.

In general, the agreement condition

$$Bel_h(A) \le Pl_h(A) \quad \forall A \in \mathcal{A}. \quad (4)$$

may not be fulfilled.

Since for $h = 0$ the condition (4) is true, the function Pl_h does not increase, and the function Bel_h does not decrease with respect to h, then there is a value $h_0 = \sup\{h : Bel_h(A) \le Pl_h(A),\ A \in \mathcal{A}\} > 0$ that the condition (4) is true on the interval $[0, h_{max})$ and false on the interval $(h_{max}, 1)$. The equalities $Pl_h(X) = 1$ and $Bel_h(\emptyset) = 0$ $\forall h \in [0, h_{max}]$ follow from (4) and the fact that $Bel_h(X) = 1$, $Pl_h(\emptyset) = 0$ $\forall h \in [0, 1)$.

The following estimates for h_{max} follow from (2) and (3).

Proposition 1. *If* $\{B \in \mathcal{A} : s(\neg A, B) \le h\} \subseteq \{B \in \mathcal{A} : s(A, B) > h\}$ $\forall A \in \mathcal{A}$, *then (4) is true.*

Corollary 1. *Inequality (4) is true if we have:*
a) $s(A,B) = \dfrac{|A \cap B|}{|B|}$ *and* $h < \dfrac{1}{2}$; *b)* $s(A,B) = \dfrac{|A \cap B|}{|X|}$ *and* $h < \dfrac{\min_{B \in \mathcal{A}} |B|}{2\,|X|}$;
c) $J(A,B) = \dfrac{|A \cap B|}{|A \cup B|}$ *and* $h < \min\limits_{B \in \mathcal{A}} \dfrac{|B|}{|X| + |B|}$; *d)* $s(A,B) = \dfrac{|A \cap B|}{\min\{|A|, |B|\}}$ *and* $h < \dfrac{1}{2}$; *e)* $s(A,B) = \dfrac{2\,|A \cap B|}{|A| + |B|}$ *and* $h < \min\limits_{B \in \mathcal{A}} \dfrac{2\,|B|}{|X| + 2\,|B|}$.

Proposition 2. *We have for measures a) or e) and for* $h \in (0,1)$*:* $Bel_h(X) = 1$,

$$Bel_h(\{x_i\}) = \sum_{B \in \mathcal{A} : x_i \in B, |B| \le \frac{1}{1-h}} m(B), \quad Pl_h(\{x_i\}) = \sum_{B \in \mathcal{A} : x_i \in B, |B| < \frac{1}{h}} m(B).$$

In addition, we have

$$Pl_h(X) = \sum_{B \in \mathcal{A} : |B| > h|X|} m(B), \quad Bel_h(\emptyset) = \sum_{B \in \mathcal{A} : |B| \le h|X|} m(B)$$

for measure a) and $Pl_h(X) = 1$, $Bel_h(\emptyset) = 0$ *for measure e).*

Example 1. Let $F = 0.2F_{\{a,b\}} + 0.3F_{\{a,c\}} + 0.4F_{\{b\}} + 0.1F_{\{a,b,c\}}$ is the body of evidence on $X = \{a, b, c\}$. Let us find the matrices **Bel** and **Pl** for the similarity measure e). We have the partition $H = \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}$ for this measure. Then

Bel =

	$[0, \frac{1}{3})$	$[\frac{1}{3}, \frac{1}{2})$	$[\frac{1}{2}, \frac{2}{3})$	$[\frac{2}{3}, 1)$
$\{a\}$	0	0	0.5	0.6
$\{b\}$	0.4	0.4	0.6	0.7
$\{c\}$	0	0	0.3	0.4
$\{a,b\}$	0.6	0.7	1	1
$\{a,c\}$	0.3	0.4	0.6	0.6
$\{b,c\}$	0.4	0.5	1	1

, **Pl** =

	$[0, \frac{1}{3})$	$[\frac{1}{3}, \frac{1}{2})$	$[\frac{1}{2}, \frac{2}{3})$	$[\frac{2}{3}, 1)$
$\{a\}$	0.6	0.5	0	0
$\{b\}$	0.7	0.6	0.4	0.4
$\{c\}$	0.4	0.3	0	0
$\{a,b\}$	1	1	0.7	0.6
$\{a,c\}$	0.6	0.6	0.4	0.3
$\{b,c\}$	1	1	0.5	0.4

.

The first columns of these matrices correspond to the classical functions Bel and Pl. Condition (4) is satisfied only for the first two columns, i.e. for $h \in [0, \frac{1}{2})$.

4 Threshold Uncertainty and Internal Conflict

Functional $U_h : \mathcal{F}(X) \to [0,1]$

$$U_h(F) = \frac{1}{2^n - 2} \sum_{A} (Pl_h(A) - Bel_h(A)), \ ; \ h \in [0, h_{max})$$

has the meaning of the uncertainty value [1] of the body of evidence $F = (\mathcal{A}, m)$ for a given threshold value $h \in [0, h_{max})$. It is easy to see that the functional $U_h(F)$ does not increase with respect to $h \in [0, h_{max})$.

The value $E_h(F) = U_0(F) - U_h(F)$ will characterize the total error in calculating the uncertainty at a given threshold value $h \in [0, h_{max})$.

Example 2. The functionals $U_h(F)$ and $E_h(F)$ for the body of evidence from example 1 are equal, respectively

$$U_h(F) = \begin{cases} \frac{13}{30}, & h \in [0, \frac{1}{3}), \\ \frac{1}{3}, & h \in [\frac{1}{3}, \frac{1}{2}). \end{cases} \quad E_h(F) = \begin{cases} 0, & h \in [0, \frac{1}{3}), \\ \frac{1}{10}, & h \in [\frac{1}{3}, \frac{1}{2}). \end{cases}$$

The internal conflict of the body of evidence $Con\_in : \mathcal{F}(X) \to [0, 1]$ characterizes the degree of unconsolidation of focal elements [8]. There are different ways to assess internal conflict. For example, the measure of internal conflict proposed in [3]:

$$Con\_in(F) = 1 - \max_{1 \le i \le n} Pl(x_i).$$

But the internal conflict of the body of evidence will be zero with respect to such a measure in any case of logical consistency of the set of focal elements: $\bigcap_{A \in \mathcal{A}} A \ne \emptyset$. This "strict" requirement for the degree of internal conflict may not always be justified. For example, if the electoral college must choose several candidates from the set $\{a, b, c, d, e\}$ and half of the electors indicated candidates $\{a, b, c\}$, and the other half indicated candidates $\{c, d, e\}$, then the measure $Con\_in(F) = 0$. However, such a body of evidence must be considered internally conflicting.

If we use the threshold plausibility function Pl_h instead of the function Pl, we obtain a threshold measure of internal conflict

$$Con\_in_h(F) = 1 - \max_{1 \le i \le n} Pl_h(x_i).$$

Since the function Pl_h does not increase, the measure of internal conflict will not decrease with increasing $h \in [0, h_{max})$. For example, the measure of internal conflict for the above example with electors and for the Jaccard similarity measure would be equal to

$$Con\_in_h(F) = 1 - \max_{1 \le i \le n} \sum_{B \in \mathcal{A}: x_i \in B, |B| < \frac{1}{h}} m(B) = \begin{cases} 0, & h \in [0, \frac{1}{3}), \\ 1, & h \in [\frac{1}{3}, \frac{1}{2}]. \end{cases}$$

Example 3. The functional $Con\_in_h(F)$ for the body of evidence from example 1 is equal to $Con\_in_h(F) = 1 - Pl_h(\{b\}) = \begin{cases} 0.3, & h \in [0, \frac{1}{3}), \\ 0.4, & h \in [\frac{1}{3}, \frac{1}{2}). \end{cases}$

Then the optimization problem of finding a threshold $h \in [0, h_{max})$ that would minimize the functional

$$\Phi_h(F) = U_h(F) + \lambda Con\_in_h(F) \to \min$$

can be formulated. The parameter $\lambda > 0$ adjusts the priority between uncertainty and internal conflict.

Example 4. The functional $\Phi_h(F)$ for the body of evidence from example 1 is equal to

$$\Phi_h(F) = \begin{cases} \frac{13}{30} + 0.3\lambda, & h \in [0, \frac{1}{3}), \\ \frac{1}{3} + 0.4\lambda, & h \in [\frac{1}{3}, \frac{1}{2}), \end{cases} \quad \arg\min_h \Phi_h(F) = \begin{cases} [0, \frac{1}{3}), & \lambda > 1, \\ [\frac{1}{3}, \frac{1}{2}), & \lambda \in (0, 1]. \end{cases}$$

The pair $(Con\_in_h(F), U_h(F))$ characterizes the "quality" of the body of evidence. Less uncertainty and less internal conflict correspond to a higher "quality" body of evidence. We can pose the problem of finding a threshold $h \in [0, h_{max}]$ at which the uncertainty will be minimal, provided that the internal conflict does not exceed a given value.

5 Threshold Aggregation and External Conflict

Suppose that two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$ are given on one and the same set X. Only strongly interacting focal elements can be taken into account when aggregating these bodies of evidence into one body of evidence $\mathcal{F}(X) \times \mathcal{F}(X) \to \mathcal{F}(X)$. Then the conjunctive threshold aggregation, similar to Dempster's rule, will take the form

$$m_h(A) = \frac{1}{k_h} \sum_{B \cap C = A, s(B,C) > h} m_1(B) m_2(C), \quad m_h(\emptyset) = 0, \tag{5}$$

where $k_h = \sum\limits_{s(B,C)>h} m_1(B) m_2(C) \neq 0$. If $k_h = 0$, then the rule (5) is not applicable. Rule (5) is a special case of Zhang's center combination rule [13]. Note that $s(B, C) > 0 \Rightarrow A = B \cap C \neq \emptyset$ in (5).

The value

$$Con_h(F_1, F_2) = 1 - k_h = \sum_{s(B,C) \leq h} m_1(B) m_2(C) \tag{6}$$

has the meaning of a threshold measure of external conflict between bodies of evidence. Measures of conflict with weights were also considered in the [9]. It is easy to see that for any similarity measure and any $F_1, F_2 \in \mathcal{F}(X)$ the following is true:

1) $Con_0(F_1, F_2) = \sum_{B \cap C = \emptyset} m_1(B) m_2(C)$ is the most popular measure of external conflict from the Dempster rule (for a review of other measures of external conflict see [7]);

2) $Con_1(F_1, F_2) = 1$.

Example 5. Let three bodies of evidence be given on $X = \{a, b, c\}$: $F_1 = 0.2F_{\{a,b\}} + 0.3F_{\{a\}} + 0.4F_{\{b\}} + 0.1F_{\{a,b,c\}}$, $F_2 = 0.6F_{\{b,c\}} + 0.4F_{\{a,b,c\}}$, $F_3 = 0.4F_{\{a\}} + 0.4F_{\{b\}} + 0.2F_{\{a,b,c\}}$. Suppose we select two bodies of evidence from these three

h	(F_1,F_2)	(F_1,F_3)	(F_2,F_3)
$\left[0,\frac{1}{3}\right)$	0.18	0.28	0.24
$\left[\frac{1}{3},\frac{1}{2}\right)$	0.58	0.44	0.56
$\left[\frac{1}{2},\frac{2}{3}\right)$	0.82	0.72	0.8
$\left[\frac{2}{3},1\right)$	0.96	0.82	0.92

Table 1. The values of the conflict measure Con_h.

with the least external conflict for subsequent aggregation. The values of the conflict measure Con_h for each pair, calculated using the formula (6) for the Jaccard similarity measure, are presented in the Table 1.

If we use the usual (threshold-free, i.e. $h=0$) measure of external conflict, then the pair F_1,F_2 will have the least conflict. But if we want to take into account (weakly) overlapping focal elements when assessing conflict, then we can use the integral characteristic of conflict. For example,

$$ICon_w(F_1,F_2)=\int\limits_0^1 w(h)Con_h(F_1,F_2)\,dh,$$

where the non-negative weight function $w(h)$ satisfies the normalization condition $\int_0^1 w(h)\,dh=1$ and regulates the priority of values $h\in[0,1)$. Small values of h should have higher priority. Therefore, the function $w(h)$ must be non-increasing.

We will get for evidence bodies from example 5 and $w(h)=1$: $ICon_w(F_1,F_2)\approx 0.613$, $ICon_w(F_1,F_3)=0.56$, $ICon_w(F_2,F_3)\approx 0.613$. In this case, choosing the pair F_1,F_3 will be preferable. If we use the weight $w(h)=\frac{3}{2}-h$, we obtain $ICon_w(F_1,F_2)\approx 0.523$, $ICon_w(F_1,F_3)\approx 0.607$, $ICon_w(F_2,F_3)\approx 0.534$. In this case, it would be preferable to choose the pair F_1,F_2 or F_2,F_3.

Integral conjunctive aggregation can be defined analogously: $im_w(A)=\int_0^1 w(h)m_h(A)\,dh$, $A\in 2^X$, where m_h are calculated, for example, by the formula (5) (if $A\in 2^X$ is not a focal element for h, then we assume $m_h(A)=0$).

6 Conclusion

The main advantage of describing bodies of evidence using threshold functions is that we can control the degree of uncertainty and conflict in such a description. In addition, the set of all represents with different thresholds gives us a more complete description of bodies of evidence and their aggregation. The problems of finding the optimal threshold at which a compromise is achieved between the accuracy of the description and uncertainty, between uncertainty and internal conflict, etc. can be posed. Some examples of such problems have been considered.

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