On Optimal Blurring of Point Expert Estimates and their Aggregation in the Framework of Evidence Theory

A. Lepskiy

Higher School of Economics, 20 Myasnitskaya Ulitsa, Moscow, 101000, Russia,
alepskiy@hse.ru   https://www.hse.ru/en/org/persons/10586209

Abstract

The article is devoted to solving the problem of choosing sources of prognostic expert information for aggregation and aggregation itself in the framework of evidence theory. A procedure for optimal "blurring" of point values and construction of evidence bodies based on the SVR regression method is proposed. A procedure for choosing predictions (bodies of evidence) for combination based on minimizing the measure of contradiction between bodies of evidence is proposed. The applicability of these procedures is demonstrated by a numerical example of selection for aggregation of forecasts of GDP growth.

Keywords. Expert Forecasts, Forecast Aggregation, Evidence Theory.

1 Introduction

Expert evaluation and forecasting is widely used in the development of programs for the socio-economic development of regions, making managerial decisions, decision making in the financial market, counteracting natural and man-made threats, etc. Aggregation of predictive information obtained from different sources is one of the important methods of expert evaluation.

There are a number of methods and algorithms for aggregation. For example, the Hedge [7] algorithm, which uses the multiplicative weights update method of Expert Advisors, is popular. Consensus forecasting as a method of averaging with some weights or using a median is widely developed in macroeconomic forecasting. However, a number of studies have shown that aggregated forecasts should be treated with caution. For example, it was argued in [9] that it is inefficient to use the mean for forecast aggregation. This inefficiency increases as the number of forecasts in the aggregation increases. Some procedures for selecting "good" forecasts for aggregation were also proposed there.

Therefore, both the problem of developing new aggregation methods and the problem of choosing "good" forecasts for aggregation, taking into account the historical data of forecasting sources, their contradictoriness remain relevant. It is convenient to model contradictoriness, uncertainty, reliability of information sources in the framework of the theory of evidence [4, 13]. Evidence theory aggregation methods were used in [15] to combine analysts' predictions about the most promising sectors of the stock market in the medium term. Some strategies for aggregating investment bank forecasts about the future value of securities were considered in [10] within the framework of evidence theory. The application of evidence theory to estimating coherence of analyst's forecasts about the cost of shares of Russian companies was studied in [2].

This article is devoted to the development of one method for constructing bodies of evidence of predictive expert estimates, taking into account historical information about forecasts, as well as aggregating these estimates in the framework of the theory of evidence.

2 Some Information from the Theory of Evidence

We will consider bodies of evidence only on the real axis $\mathbb{R}$. Therefore, we will give the necessary information from the theory of evidence [4, 13] in relation to this case. Let X be some interval of the real axis $\mathbb{R}$, $\mathcal{A}_X$ be the set of all segments on X, and $\mathcal{A}$ be a finite subset of the set $\mathcal{A}_X$. Some non-negative set function $m : \mathcal{A}_X \to [0,1]$ is defined on $\mathcal{A}_X$ and satisfies the normalization condition $\sum_{A\in\mathcal{A}} m(A) = 1$. Without loss of generality, we will assume that $m(A) > 0 \Leftrightarrow A \in \mathcal{A}$. In this case, $\mathcal{A}$ is called the set of focal elements, m is the mass function, and the pair $F = (\mathcal{A}, m)$ is the body of evidence on X. The evidence body provides information about the degree of belief that the true alternative belongs to any focal element. Special cases of evidence bodies are: 1) categorical evidence $F_A = (A, 1)$; 2) vacuous evidence $F_X = (X, 1)$. An arbitrary body of evidence $F = (\mathcal{A}, m)$ can be represented as $F = \sum_{A\in\mathcal{A}} m(A)F_A$. A body of evidence of the form $F_A^m = mF_A + (1-m)F_X$, $m \in [0,1]$ is called simple.

In the theory of evidence, in particular, the problems of analyzing the contradiction of information from different sources [11] and aggregating such information are considered. Let two independent sources of information be described by two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$. To assess the contradiction (conflict) between the evidence bodies F_1 and F_2, we will use the measure

$$Con(F_1, F_2) = \sum_{A\in\mathcal{A}_1, B\in\mathcal{A}_2} \gamma(A,B)m_1(A)m_2(B), \tag{1}$$

where $\gamma(A,B) = 1 - s(A,B)$ and $s(A,B)$ is the similarity coefficient satisfying the conditions: 1) $0 \le s(A,B) \le 1$; 2) $s(A,B) = 0$, if $A \cap B = \emptyset$; 3) $s(A,A) = 1$. Jaccard index $s(A,B) = L(A\cap B)/L(A\cup B)$ is an example of such a coefficient which we will use below. Here $L(C)$ is the length of the segment $C \subseteq \mathbb{R}$ (the sum of the lengths if C is the union of several segments).

The conflict measure (1) is associated with the following conjunctive rule for aggregating evidence bodies $F_1 \otimes F_2 = (\mathcal{A}, m)$, where $\mathcal{A} = \{C = A\cap B : \ A \in \mathcal{A}_1, B \in \mathcal{A}_2\}$

$$m(C) = \frac{1}{K} \sum_{A\cap B = C} s(A,B)m_1(A)m_2(B), \tag{2}$$

if $K = 1 - Con(F_1, F_2) \ne 0$. This is the so-called Zhang's center combination rule [16].

3 Finding Evidence Bodies of Experts' Forecasts

Let $x(t)$ be some time dependent indicator. For example, it can be a macroeconomic indicator (GDP, oil cost, inflation) or the area of forest fires in the region. Let's assume that some expert at the time t_{i-1} made a prediction f_i about the values of this indicator at the time t_i and $x_i = x(t_i)$ is the real value of the indicator at the moment of time t_i, $i = 1, \dots, N$. As a result, we have a sample of predictive values $\{f_i\}_{i=1}^N$ and a sample of real values $\{x_i\}_{i=1}^N$ of the indicator for N previous time points. We consider that $t_1 < \dots < t_N$.

Let's assume that the expert at the time t_N made a prediction f_{N+1} about the values of the indicator at the time t_{N+1}. It is necessary to form a body of evidence that would reflect the information about the forecast, taking into account the historical information about the previous forecasts of this expert.

We will find the body of evidence in the form of a simple belief structure

$$F_{[l,r]}^m = mF_{[l,r]} + (1-m) F_X, \tag{3}$$

where $[l, r]$ is the interval containing the true value of the indicator with a degree of belief $m \in [0, 1]$, X is the set all possible values of the indicator.

The interval $[l, r]$ will be obtained from the forecast value f_{N+1} by its "blur" taking into account historical information about previous forecasts. The degree of belief m will be obtained from the estimate of the reliability of the forecast.

3.1 Statistics of Biases of Expert Estimates

Knowing the samples $\{f_i\}_{i=1}^N$ and $\{x_i\}_{i=1}^N$ you can get the sample $\Delta = \{\delta_i\}_{i=1}^N$ absolute biases of forecast values

$$\delta_i = x_i - f_i, \ \ i = 1, \dots, N$$

for dimensionless indicators (percentage of GDP growth, percentage of inflation, etc.) or a sample of relative biases $\delta_i = \dfrac{x_i - f_i}{f_i}$, $i = 1, \dots, N$.

Below we consider only the case of dimensionless indicators and absolute biases. The case of relative biases is considered similarly.

In addition, we will consider the sample mean bias $\overline{\delta}$ and the sample mean square deviation s of the bias:

$$\overline{\delta} = \frac{1}{N}\sum_{i=1}^{N}\delta_i, \quad s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\delta_i - \overline{\delta}\right)^2}.$$

3.2 Formation of Focal Elements and Estimation of their Masses

One can construct an appropriate confidence interval using the Student's distribution for the "blur" point values $\{f_i\}_{i=1}^N$.

But below we will consider another approach, which can be called optimization. We will find segments $[l_i, r_i]$, $i = 1, \dots, N$ in the form

$$\begin{cases} l_i = f_i + \overline{\delta} - as, \\ r_i = f_i + \overline{\delta} + bs, \end{cases} \tag{4}$$

where the values $a, b \geq 0$ are found from the condition

$$x_i \in [l_i, r_i] \Leftrightarrow \tilde{x}_i = x_i - f_i - \overline{\delta} \in [-as, bs], \ i = 1, \ldots, N \tag{5}$$

and conditions for minimizing a certain criterion. For example, the length of the segment $[l_i, r_i]$ can be chosen as a criterion (all segments are equal in length)

$$L(a, b) = r_i - l_i = (a + b) s \to \min. \tag{6}$$

It is easy to see that the solution of the problem (5), (6) will be the boundaries

$$a_0 = \max\left\{0, \max_{1 \leq i \leq N}\left\{\frac{\overline{\delta} - \delta_i}{s}\right\}\right\}, \quad b_0 = \max\left\{0, \max_{1 \leq i \leq N}\left\{\frac{\delta_i - \overline{\delta}}{s}\right\}\right\}. \tag{7}$$

Note that the condition (5) in the optimization approach is rigid. Therefore, such a blur method will be highly unstable to data outliers. The robustness of the optimization algorithm can be achieved by using the "soft" condition instead of the condition (5)

$$l_i - \xi_i^- \leq x_i \leq r_i + \xi_i^+, \quad i = 1, \ldots, N, \tag{8}$$

where the auxiliary variables $\xi_i^- \geq 0$, $\xi_i^+ \geq 0$ have the meaning of the values that go beyond the boundaries of the interval $[l_i, r_i]$, $i = 1, \ldots, N$.

Then the problem of "soft" optimization of finding the boundaries of blur intervals $[l_i, r_i]$, $i = 1, \ldots, N$ can be formulated as follows. It is necessary to find the values a, b, $\boldsymbol{\xi}^- = (\xi_i^-)$, $\boldsymbol{\xi}^+ = (\xi_i^+)$ that would satisfy the conditions (4), (8) and minimize the functional

$$G\left(a, b, \boldsymbol{\xi}^-, \boldsymbol{\xi}^+\right) = L(a, b) + c \sum_{i=1}^{N} \left(\xi_i^- + \xi_i^+\right), \tag{9}$$

where the summand $\sum_{i=1}^{N} \left(\xi_i^- + \xi_i^+\right)$ characterizes the penalty for the total error of forecast values going beyond the intervals $[l_i, r_i]$, $i = 1, \ldots, N$, $c \geq 0$ is a parameter that regulates the relationship between the minimization of the length of the segment $L(a, b)$ and the penalty for not falling into this segment.

Note that this "soft" optimization method is similar to the well-known SVM regression method [5].

The mass function should reflect the degree of belonging of the true value to the forecast interval. Therefore, we will find the mass function as the relative frequency of hitting the real values x_i of the indicator in the intervals $[l_i, r_i]$, $i = 1, \ldots, N$:

$$m = \frac{|i : x_i \in [l_i, r_i], \ \ i = 1, \ldots, N|}{N} = \frac{|i : \tilde{x}_i \in [-as, bs], \ \ i = 1, \ldots, N|}{N}. \tag{10}$$

3.3 Optimization of the Procedure for Finding the Body of Evidence

We will get a body of evidence of the form (3) as a result of the "blur" predictive value of f_{N+1} and the mass estimate. In this case, the body of evidence F will depend on only one parameter $c \geq 0$: $F = F(c)$, which can be estimated by introducing additional restrictions. Let us consider the minimization of the imprecision measure of information given by the body of evidence F as such a constraint. There are different ways of estimating the imprecision of evidence bodies (see, for example, [1]). Below we will use the generalized cardinality as a imprecision measure, which was introduced in [6]. This measure was defined as $H(F) = \sum_{A\in\mathcal{A}} m(A)\,|A|$ for the evidence body $F = (\mathcal{A}, m)$ on a finite base set X. We will consider the length of the focal element-segment $L(A)$ instead of the cardinality $|A|$ on the base set $X \subseteq \mathbb{R}$.

The length of the optimal interval $L(c) = (a + b)\,s$ does not decrease as the parameter $c \geq 0$ increases, which follows from (9). In particular, $L(c) \to L_{max} = (a_0 + b_0)\,s$ as $c \to \infty$ ("hard" optimization) and $L(c) \to 0$ for $c \to 0$.

If we assume that the sample points $\{\tilde{x}_i\}$ are distributed according to the normal law $N\left(\overline{\delta}, s^2\right)$, then

$$m \approx P\left\{\theta \in [-as, bs]\right\} = \Phi(a) + \Phi(b),$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}}\,dt$ is the Laplace function. Then the imprecision measure $H(F)$ will be approximately equal to

$$H(F) = (\Phi(a) + \Phi(b))\,(a + b)\,s + (1 - \Phi(a) - \Phi(b))\,L(X).$$

It can be shown that this function achieves a global minimum at the point (a_{opt}, b_{opt}), where $a = a_{opt} = b_{opt}$ is the only root of the equation

$$2\Phi(a)s = \Phi'(a)\,(L(X) - 2as)\,,\ \text{ if } 0 \leq 2as \leq L(X).$$

The minimum point (a_{opt}, b_{opt}) will correspond to the optimal value of the parameter $c_{opt} = L^{-1}((a_{opt} + b_{opt})s)$. In practice, the optimal value of c_{opt} can be found by constructing a graph of the $H(F(c))$ dependence.

4 Selection and Aggregation of Expert Assessments

Assume that there are M predictions $f_{N+1}^{(k)}$, $k = 1, \ldots, M$ about the values of the considered indicator at the time t_{N+1} and historical information about past forecasts obtained from various independent sources. Let's find evidence bodies F_k, $k = 1, \ldots, M$ about a new forecast for each source based on this information. Multiple evidence bodies are required to be selected for aggregation. The analysis of information sources for conflict is one of the main methods for deciding about the choice of sources for aggregation.

General Scheme for Aggregating Expert Forecasts

1. Find the sample mean biases $\overline{\delta}^{(k)}$, and the sample mean square deviation $s^{(k)}$ (see subsection 3.1) , the parameters of the boundaries of the optimal intervals $a_{opt}^{(k)}$ and $b_{opt}^{(k)}$ (i. e. values of a and b at the optimal value of c_{opt}), mass functions m_k (see subsections 3.2 and 3.3) for each k-th source of information $k = 1, \ldots, M$.

2. Find simple evidence bodies of a new prediction of the form $F_k = F^{m_k}_{[l_k, r_k]}$ for each k-th predictive value $f^{(k)}_{N+1}$, $k = 1, \ldots, M$, where

$$\begin{cases} l_k = f^{(k)}_{N+1} + \overline{\delta}^{(k)} - a^{(k)}_{opt} s^{(k)}, \\ r_k = f^{(k)}_{N+1} + \overline{\delta}^{(k)} + b^{(k)}_{opt} s^{(k)}, \end{cases}$$

$X = con \left\{ \bigcup_{k=0}^{M} [l_k, r_k] \right\}$, con is the minimal convex hull.

3. Find the values of pairwise conflicts $Con(F_k, F_p)$, $k = 1, \ldots, M-1$, $p = k+1, \ldots, M$ by the formula (1).

4. Find the aggregation of a pair of evidence bodies with minimal conflict into a new evidence body using the formula (2).

5 Numerical Example

Data description. Let us apply the above-described method of forming bodies of evidence and selecting sources of information for aggregation using the example of forecasts on the growth rate of Russia's GDP in 2010-2019. The sources of forecasts are [3] : 1) The International Monetary Fund (IMF); 2) The Centre of Development Institute – HSE University (HSE); 3) The Ministry of Economic Development of the Russian Federation (MED). Forecast and real values of the GDP growth rate are given in Table. 1, and the corresponding graphs are shown in fig. 1.

Table 1: Russia's GDP Growth Rate and Forecasts in 2010-2019.

	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
GDN growth	4,5	4,3	4,0	1,8	0,7	-2,0	0,2	1,8	2,8	2,2
IMF	1,5	4,8	3,5	3,8	2,0	0,5	-1,0	1,0	1,6	1,6
HSE	2,6	4,0	3,5	3,2	2,1	0,2	-0,7	1,0	1,7	1,4
MED	1,6	4,2	3,4	3,7	3,0	1,2	-0,6	1,0	2,1	1,3

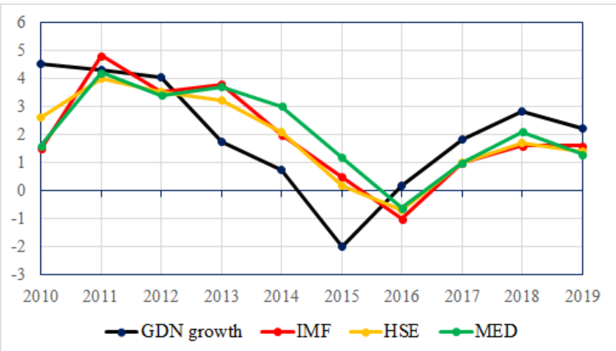

Figure 1: Real GDP and forecasts, % growth

Construction of bodies of evidence-forecasts. The bodies of evidence for the 2019 forecasts are found from the 2010–2018 data and using the methodology described in the section 3.

The segment $X = [-3,5; 3,5]$ is selected as the base interval at the "blur" stage. This segment contains all $\{\tilde{x}_i\}$ forecast points (after removal of the trend and average bias) of all sources.

Graphs of dependences of the imprecision measure $H(F)$ of forecasts on the values of the parameter c for different sources are shown in fig. 2.

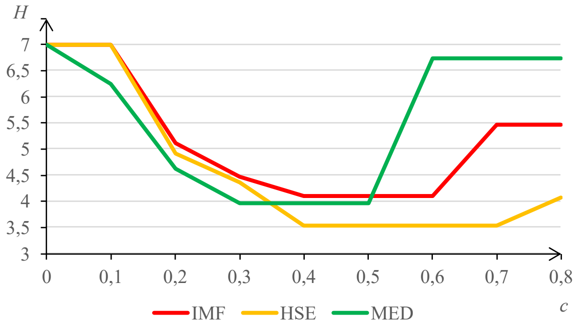

Figure 2: Dependences of the imprecision measure $H(F)$ on the parameter c

The results of "hard" and "soft" optimization "blur" point forecasts for 2019 and mass estimation are given in Table. 2. Here $[-a_0 s, b_0 s]$ and $[-a_{opt} s, b_{opt} s]$ are the "hard" and "soft" segments optimal blurs of the point value $\tilde{x}_N$ (after removal of the trend and average bias), respectively.

Table 2: "Blur" results of 2019 point forecasts and mass estimates.

	$\bar{\delta}$	s	$-a_0 s$	$b_0 s$	$-a_{opt} s$	$b_{opt} s$	m	H	c_{opt}
IMF	0,05	1,16	–2,53	2,95	–2,1	1,16	0,78	4,08	$0,4 \div 0,6$
HSE	0,06	1,3	–2,24	1,84	–1,51	1,04	0,78	3,54	$0,4 \div 0,7$
MED	–0,16	1,81	–3,01	3,06	–2,11	1,0	0,78	3,96	$0,3 \div 0,5$

Thus, we have three predictions in the form of bodies of evidence:

$$F_1 = 0,78 F_{[-0,45;2,8]} + 0,12 F_X, \quad F_2 = 0,78 F_{[-0,05;2,5]} + 0,12 F_X,$$

$$F_3 = 0,78 F_{[-0,96;2,12]} + 0,12 F_X,$$

which we will now consider on the base set $X = [-1; 3]$.

Forecast aggregation. Let us apply the general scheme of forecast aggregation. The pairwise conflict matrix $Con = \{Con\,(F_k, F_p)\}$ will be equal to

$$Con = \begin{pmatrix} 0,064 & 0,226 & 0,265 \\ 0,226 & 0,125 & 0,329 \\ 0,265 & 0,329 & 0,079 \end{pmatrix}.$$

The smallest conflict is between the IMF and HSE forecasts. But we find the aggregation of all three pairs. We'll get (1 – IMF, 2 – HSE, 3 – DEM):

$$F_{12} = 0,76F_{[-0,05;2,5]} + 0,18F_{[-0,45;2,8]} + 0,06F_X,$$

$$F_{13} = 0,56F_{[-0,45;2,12]} + 0,19F_{[-0,45;2,8]} + 0,18F_{[-0,96;2,12]} + 0,07F_X,$$

$$F_{23} = 0,56F_{[-0,05;2,12]} + 0,16F_{[-0,05;2,5]} + 0,2F_{[-0,96;2,12]} + 0,08F_X.$$

Let's find the change in the distance between the actual value of the indicator and the predicted values before aggregation and after aggregation. To do this, we will use the conflict measure $Con(F_\tau, F)$ between the body of evidence-prediction $F = (\mathcal{B}, m)$ and the categorical evidence F_τ, which equals the $\tau-$blur of the real value x_{N+1} (i.e. $F_\tau = F_{U_\tau(x_{N+1})}$), where $U_\tau(x) = [x - \tau, x + \tau]$, $\tau > 0$ is some parameter. We have

$$Con(F_\tau, F) = 1 - \sum_{B \in \mathcal{B}} s\left(U_\tau(x_{N+1}), B\right) m(B).$$

If $x_{N+1} \notin \partial B \ \forall B \in \mathcal{B}$ and $\tau > 0$ such that $U_\tau(x_{N+1}) \subsetneqq B$, if $x_{N+1} \in B$, then

$$Con(F_{U_\tau(x_{N+1})}, F) = 1 - 2\tau P_F(x_{N+1}),$$

where $P_F(x) = \sum_{B \in \mathcal{B}: x \in B} \dfrac{m(B)}{\mid B \mid} = \dfrac{1}{2\tau}\left(1 - Con(F_{U_\tau(x)}, F)\right)$ is the consistency density of predictive evidence and real value evidence. Note that this value has the meaning of the so-called pignistic probability [14] for a finite set X. Therefore, the value $P_F(x_{N+1})$, which is independent of "blur" $\tau > 0$, will be used as the degree of closeness of the evidence-forecast body $F = (\mathcal{B}, m)$ to the real value of x_{N+1}.

In addition, we find the change in the imprecision measure H for estimating the quality of aggregation.

The values of $P_F(x_{N+1})$ and $H(F)$ for all evidence bodies F_1, F_2, F_3 and their aggregations $F_1 \otimes F_2$, $F_1 \otimes F_3$, $F_2 \otimes F_3$ are given in the Table 3.

Table 3: Values of consistency density and imprecision measure.

	F_1	F_2	F_3	$F_1 \otimes F_2$	$F_1 \otimes F_3$	$F_2 \otimes F_3$
$P_F(x_{N+1})$	0,294	0,361	0,055	**0,368**	0,075	0,083
$H(F)$	3,42	2,87	3,28	2,72	2,89	**2,54**

It is easy to see that after aggregation, the proximity of the forecast to the exact value slightly increased relative to the P_F measure and is achieved by combining the least conflicting pair of evidence bodies F_1 and F_2. In addition, the accuracy of forecasts after combining all pairs also increased.

6 Conclusion

The following results were obtained in the article:

- a procedure for optimal "blurring" of point values and construction of evidence bodies based on the SVR regression method is proposed, taking into account the historical prognostic information of the source;

- a procedure for finding the optimal values of the parameters of the ¡¡blur¿¿ procedure based on minimizing the measure of inaccuracy of the resulting body of evidence is proposed;
- a procedure for selecting predictions (evidence bodies) for combination based on minimizing the conflict measure between evidence bodies is proposed;
- the applicability of these procedures is demonstrated by the example of selection for aggregation of GDP growth forecasts.

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