Threshold Aggregation of Fuzzy Data Using Fuzzy Cardinalities

Alexander Lepskiy

National Research University – Higher School of Economics,
Moscow, Russia

INFUS 2024, July 16–18, 2024, Çanakkale, Turkey

Research Motivation

- Let us consider the classical problem of aggregation of individual preferences. There are many rules for such aggregation.
- In some cases, aggregation rules should be **non-compensatory**. This implies that low scores on one criterion cannot be compensated for by high scores on others.
- The so-called **threshold aggregation** (**TA**) rule [Aleskerov et al 2010, Aleskerov & Yakuba 2007] is one of the popular aggregation rules that has a non-compensatory property.
- In some cases, the characteristics of alternatives may be inaccurate. Then the problem of generalizing the TA rule to the case of inaccurate data is relevant.

Outline of Presentation

- Threshold Aggregation (TA) Problem with Accurate Data;
- TA Problem with Inaccurate Data;
- The procedure for Finding the Fuzzy Cardinality (FC) of a Set of Three-Grade Fuzzy Estimates;
- Comparison of FC and Ranking of Alternatives;
- Numerical Example;
- Summary and Conclusion.

TA Problem with Accurate Data

The problem of ranking alternatives of a set X of evaluated by n criteria in a **three-gradation scale** is being considered. The alternatives are represented by vectors: $\mathbf{x} = (x_1, \ldots, x_n)$, where $x_i \in \{1, 2, 3\}$. It is required to find an operator $\varphi_n = \varphi : X \to \mathbb{R}$ that satisfies the conditions [Aleskerov & Yakuba 2007]:

1) **Pareto-domination**:
   if $\mathbf{x}, \mathbf{y} \in X$ and $x_i \geq y_i \quad \forall i, \exists s : \; x_s > y_s$, then $\varphi(\mathbf{x}) > \varphi(\mathbf{y})$;
2) **pairwise compensability of criteria**:
   if $\mathbf{x}, \mathbf{y} \in X$ and $v_k(\mathbf{x}) = v_k(\mathbf{y})$ $k = 1, 2$, then $\varphi(\mathbf{x}) = \varphi(\mathbf{y})$, where $v_k(\mathbf{x}) = |\{i : x_i = k\}|$ is the number of estimates of k in the alternative $\mathbf{x}$, $k = 1, 2, 3$;
3) **threshold noncompensability**:
   $\varphi(\underbrace{2, \ldots, 2}_{n}) > \varphi(\mathbf{x})$ $\forall \mathbf{x} \in X$: $\exists s : \; x_s = 1$;

4) **the reduction axiom**:
if $\forall \mathbf{x}, \mathbf{y} \in X\ \exists s:\ x_s = y_s$, then
$\varphi_n(\mathbf{x}) > \varphi_n(\mathbf{y}) \Leftrightarrow \varphi_{n-1}(\mathbf{x}_{-s}) > \varphi_{n-1}(\mathbf{y}_{-s})$,
where $\mathbf{x}_{-s} = (x_1, \ldots, x_{s-1}, x_{s+1}, \ldots, x_n)$.

It is shown that the **lexicographic aggregation rule** is a solution to this problem:

$$\varphi(\mathbf{x}) > \varphi(\mathbf{y}) \Leftrightarrow$$

$$v_1(\mathbf{x}) < v_1(\mathbf{y}) \ \text{ or}$$

$$\exists j \in \{1, 2\} : v_k(\mathbf{x}) = v_k(\mathbf{y}) \quad \forall k \leq j \ \text{ and } \ v_{k+1}(\mathbf{x}) < v_{k+1}(\mathbf{y}).$$

This problem was generalized in [Aleskerov et al 2010] to the case of m-gradation scales, $m \geq 3$.

TA Problem with Inaccurate Data

There are two main approaches to generalizing TA in the case of inaccurate data:

- development of axiomatics and aggregation rules based on this new axiomatics;
- finding an analogue of the cardinality vector of estimates for inaccurate data.

An analogue of the cardinality vector of estimates for inaccurate data can be found in two ways:

- finding the vector of scalar cardinalities of a set of inaccurate estimates;
- finding the inaccurate cardinality function of a set of inaccurate estimates.

The nature of inaccuracies in data can be different: interval, fuzzy, probabilistic, etc.

Bellow we will only talk about fuzzy models of data inaccuracy.

Let the alternatives be represented by n-dimensional vectors $\widetilde{\mathbf{x}} = (\widetilde{x_1}, \ldots, \widetilde{x_n})$ of fuzzy sets. Each fuzzy set $\widetilde{x_i}$ is defined on a three-gradation base set $\{L, M, H\}$ and has a membership function $\mu_{\widetilde{x_i}}$. We will represent a fuzzy set $\widetilde{x_i}$ as a vector of values of its membership function: $\widetilde{x_i} = (\mu_{\widetilde{x_i}}(L), \mu_{\widetilde{x_i}}(M), \mu_{\widetilde{x_i}}(H))$.

The first approach (let's call it **scalar** [Lepskiy 2023]) is based on calculating a certain measure of proximity $F_S(\widetilde{x}_i) = \psi(d(\widetilde{x}_i, S_0))$ between all fuzzy estimates $\widetilde{x}_i$ of a certain gradation $S \in \{L, M, H\}$ and the reference estimate S_0 for a given gradation.

Then the value

$$v_S(\widetilde{\mathbf{x}}) = \sum_{\widetilde{x}_i \in S} F_S(\widetilde{x}_i), \quad S \in \{L, M, H\}$$

characterizes the cardinality of the set of fuzzy estimates of the class S.

Now I'll consider another approach, which is based on calculating the **fuzzy cardinality (FC)** of a set of fuzzy estimates of each gradation.

The procedure for Finding the FC of a Set of Three-Grade Fuzzy Estimates

Fuzzy cardinality (FC) $\widetilde{v_S}(\widetilde{\mathbf{x}})$ will be defined on the base set $\{0,\ldots,n\}$ (n is the number of criteria). The FC membership function $\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}$ of the class $S\in\{L,M,H\}$ must satisfy the following conditions:

1) $\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(k)=1 \Leftrightarrow k=\left\lfloor \sum_{\widetilde{x}\in S_{\widetilde{\mathbf{x}}}}\mu_{\widetilde{x}}(S)\right\rfloor$, where $\lfloor\ \rfloor$ is rounding down, $S_{\widetilde{\mathbf{x}}}$ is a set of estimates of class S for which the maximum membership function is achieved;
2) $\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(k)=0$, if $k<\left\lfloor \sum_{\widetilde{x}\in S_{\widetilde{\mathbf{x}}}}\mu_{\widetilde{x}}(S)\right\rfloor$.
This condition means that the cardinality of the set of estimates for a class S cannot be less than the number of estimates that obviously belong to this class.

Desirable properties of FC would also be the following.
Let Fuz is a certain degree of fuzziness of the set and

$$Fuz\left(\widetilde{\mathbf{x}}\right) = \left(Fuz\left(\widetilde{x_1}\right), \ldots, Fuz\left(\widetilde{x_n}\right)\right).$$

For $\mathbf{a} = (a_1, \ldots, a_n)$ and $\mathbf{b} = (b_1, \ldots, b_n)$ vectors, comparison $\mathbf{a} \geq \mathbf{b}$ means that $a_1 \geq b_1, \ldots, a_n \geq b_n$.

3) if $Fuz\left(\widetilde{\mathbf{x}}\right) \geq Fuz\left(\widetilde{\mathbf{y}}\right)$, then $Fuz\left(\widetilde{v_S}(\widetilde{\mathbf{x}})\right) \geq Fuz\left(\widetilde{v_S}(\widetilde{\mathbf{y}})\right)$ $\forall S \in \{L, M, H\}$.
4) if $Fuz\left(\widetilde{\mathbf{x}}\right) = \mathbf{0}$, then $\widetilde{v_S}(\widetilde{\mathbf{x}}) = v_S(\mathbf{x})$ $\forall S \in \{L, M, H\}$.

   The last condition means that if all fuzzy estimates are non-fuzzy (i. e. $\mu_{\widetilde{x_i}}(S) \in \{0, 1\}$ $\forall S \in \{L, M, H\}$), then the FC of the vector estimate will coincide with the usual cardinality.

We will find the remaining values $\mu_{\widetilde{v_S(\widetilde{\mathbf{x}})}}(k)$ for $k > \left\lfloor \sum_{\widetilde{x} \in S_{\widetilde{\mathbf{x}}}} \mu_{\widetilde{x}}(S) \right\rfloor$ using the following **threshold rule**.

Let $S_1, S_2, S_3 \in \{L, M, H\}$ be three different classes of estimates such that $\mu_{\widetilde{x_i}}(S_1) > \mu_{\widetilde{x_i}}(S_2) \geq \mu_{\widetilde{x_i}}(S_3)$.

Then we will call the estimate $\widetilde{x_i}$ the $\mathbf{1}^{st}$ **level** estimate for the class S_1, the $\mathbf{2}^{nd}$ **level** for the class S_2 and the $\mathbf{3}^{rd}$ for the class S_3.

We will order all values $\mu_{\widetilde{x_i}}(S) = q_i$, $i = 1, \ldots, n$ in ascending order of level numbers for a fixed class S:

$$q_{i_1}^{(1)}, \ldots, q_{i_k}^{(1)}, q_{i_k+1}^{(2)}, \ldots, q_{i_r}^{(2)}, q_{i_r+1}^{(3)}, \ldots, q_{i_n}^{(3)}$$

(the superscript is the level number).

Then we get for the $\mathbf{1}^{st}$ **level** values, according to condition 1):
$\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(p_1) = 1$, where $p_1 := \left\lfloor q_{i_1}^{(1)} + \ldots + q_{i_k}^{(1)} \right\rfloor$. If there are no 1^{st} level estimates, then we assume $p_1 = 0$.

If there are quite a lot of large $\mathbf{2}^{nd}$ **level** values, then this means that the values of the FC membership function will be quite large for cardinalities greater than p_1. For example, the following threshold procedure may be proposed.

If $p_2 = \left\lfloor \left\{ q_{i_1}^{(1)} + \ldots + q_{i_k}^{(1)} \right\} + q_{i_k+1}^{(2)} + \ldots + q_{i_r}^{(2)} \right\rfloor \geq 1$, then
$\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(p_1 + 1) = \ldots = \mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(p_1 + p_2) = m_2$, where $m_2 \in (0, 1)$. Here $\{\ \}$ is the fractional part of the number.

The values of the $\mathbf{3}^{rd}$ **level** are taken into account in the same way. If $p_3 = \left\lfloor \left\{ \left\{ q_{i_1}^{(1)} + \ldots + q_{i_k}^{(1)} \right\} + q_{i_k+1}^{(2)} + \ldots + q_{i_r}^{(2)} \right\} + q_{i_r+1}^{(3)} + \ldots + q_{i_n}^{(3)} \right\rfloor \geq 1$, then we will increase by m_3 the membership function $\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}$ for the values of the argument $p_1 + 1, \ldots, p_1 + p_3$, where
$0 < m_3 < \min\{m_2, 1 - m_2\}$.

Example

Let the vector $\widetilde{\mathbf{x}} = (\widetilde{x_1}, \ldots, \widetilde{x_5})$ from 5 fuzzy estimates be given, where $\widetilde{x_i} = (\mu_{\widetilde{x_i}}(L), \mu_{\widetilde{x_i}}(M), \mu_{\widetilde{x_i}}(H))$, $i = 1, \ldots, 5$ and

$$\widetilde{x_1} = (0.5,\ 0.6,\ 1),\ \ \widetilde{x_2} = (0.3,\ 0.5,\ 1),$$

$$\widetilde{x_3} = (0.5,\ 1,\ 0.4),\ \ \widetilde{x_4} = (1,\ 0.8,\ 0.3),\ \ \widetilde{x_5} = (1,\ 0.5,\ 0.2).$$

Then we will get the following results of calculating the values of the FC membership function for each class

$$\widetilde{v_L}(\widetilde{\mathbf{x}}) = (0, 0, 1, m_3, 0, 0),\ \ \widetilde{v_M}(\widetilde{\mathbf{x}}) = (0, 1, m_2, m_2, 0, 0),$$

$$\widetilde{v_H}(\widetilde{\mathbf{x}}) = (0, 0, 1, 0, 0, 0).$$

Here m_2, m_3 are some threshold values that satisfy the conditions $m_2 \in (0, 1)$, $0 < m_3 < \min\{m_2, 1 - m_2\}$.

Comparison of FC and Ranking of Alternatives

Let $\mathcal{V}_n$ be the set of all FCs for n fuzzy estimates.

To apply the lexicographic rule for ranking a set of vector fuzzy alternatives $\{\widetilde{\mathbf{x}}\}$ with respect to FC $\widetilde{\mathbf{v}}(\widetilde{\mathbf{x}}) = (\widetilde{v_L}(\widetilde{\mathbf{x}}), \widetilde{v_M}(\widetilde{\mathbf{x}}), \widetilde{v_H}(\widetilde{\mathbf{x}}))$, it is necessary to use some rule for ordering fuzzy sets. This can be done using some defuzzification function $F : \mathcal{V}_n \to \mathbb{R}$.

We will assume that the FCs of class S estimates are in the relation $\widetilde{v_S}(\widetilde{\mathbf{x}}) \prec \widetilde{v_S}(\widetilde{\mathbf{y}})$ for two alternatives $\widetilde{\mathbf{x}}$ and $\widetilde{\mathbf{y}}$ if $F(\widetilde{v_S}(\widetilde{\mathbf{x}})) < F(\widetilde{v_S}(\widetilde{\mathbf{y}}))$ and are equal $\widetilde{v_S}(\widetilde{\mathbf{x}}) \sim \widetilde{v_S}(\widetilde{\mathbf{y}})$ if $F(\widetilde{v_S}(\widetilde{\mathbf{x}})) = F(\widetilde{v_S}(\widetilde{\mathbf{y}}))$.

For example, if we use center of gravity

$$G(\widetilde{v_S}(\widetilde{\mathbf{x}})) = \sum_{i=0}^{n} i\mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(i) \Big/ \sum_{i=0}^{n} \mu_{\widetilde{v_S}(\widetilde{\mathbf{x}})}(i)$$

as the defuzzification function, we get for the example above:

$$G\left(\widetilde{v_L}(\widetilde{\mathbf{x}})\right) = \frac{2+3m_3}{1+m_3}, \;\; G\left(\widetilde{v_M}(\widetilde{\mathbf{x}})\right) = \frac{1+5m_2}{1+2m_2}, \;\; G\left(\widetilde{v_H}(\widetilde{\mathbf{x}})\right) = 2.$$

For any acceptable threshold values m_2 and m_3, we have $G\left(\widetilde{v_M}(\widetilde{\mathbf{x}})\right) < G\left(\widetilde{v_H}(\widetilde{\mathbf{x}})\right) < G\left(\widetilde{v_L}(\widetilde{\mathbf{x}})\right)$. Therefore, the ranking $\widetilde{v_M}(\widetilde{\mathbf{x}}) \prec \widetilde{v_H}(\widetilde{\mathbf{x}}) \prec \widetilde{v_L}(\widetilde{\mathbf{x}})$ is correct.

Another way to compare the FC of sets of fuzzy estimates is to use the same **lexicographic rule**.
Let $\widetilde{v_S}\left(\widetilde{\mathbf{x}}\right) = (a_0, \ldots, a_n)$, $\widetilde{v_S}\left(\widetilde{\mathbf{y}}\right) = (b_0, \ldots, b_n)$.

Then we will assume that $\widetilde{v_S}\left(\widetilde{\mathbf{x}}\right) \prec \widetilde{v_S}\left(\widetilde{\mathbf{y}}\right)$ if $a_0 < b_0$ or $\exists k \in \{0, \ldots, n-1\} : a_0 = b_0, \ldots, a_k = b_k, a_{k+1} < b_{k+1}$.

Otherwise, we assume that $\widetilde{v_S}\left(\widetilde{\mathbf{x}}\right) \sim \widetilde{v_S}\left(\widetilde{\mathbf{y}}\right)$.

Now, the threshold aggregation rule for two alternatives $\widetilde{\mathbf{x}}$ and $\widetilde{\mathbf{y}}$ will be as follows:

$$\begin{aligned}\varphi(\widetilde{\mathbf{x}}) > \varphi(\widetilde{\mathbf{y}}) \Leftrightarrow \quad & \widetilde{v_L}(\widetilde{\mathbf{x}}) \prec \widetilde{v_L}(\widetilde{\mathbf{y}}) \\ & \text{or } \widetilde{v_L}(\widetilde{\mathbf{x}}) \sim \widetilde{v_L}(\widetilde{\mathbf{y}}),\ \widetilde{v_M}(\widetilde{\mathbf{x}}) \prec \widetilde{v_M}(\widetilde{\mathbf{y}}) \\ & \text{or } \widetilde{v_L}(\widetilde{\mathbf{x}}) \sim \widetilde{v_L}(\widetilde{\mathbf{y}}),\ \widetilde{v_M}(\widetilde{\mathbf{x}}) \sim \widetilde{v_M}(\widetilde{\mathbf{y}}),\ \widetilde{v_H}(\widetilde{\mathbf{x}}) \prec \widetilde{v_H}(\widetilde{\mathbf{y}}).\end{aligned}$$

Numerical Example

Consider an example of ranking articles of conferences in the conference management system, such as EasyChair.

This system uses a septennial scoring system $x_i \in \{-3, -2, -1, 0, 1, 2, 3\}$, corresponding to the recommendations "strong reject", "reject", "weak reject", "borderline paper", "weak accept", "accept", "strong accept". In addition, the reviewer gives an assessment on a five-fold scale (0.2 – "none", 0.4 – "low", 0.6 – "medium", 0.8 – "high", 1 – "expert") about the degree of confidence in the correctness of his decision: $\lambda_i \in \{0.2, 0.4, 0.6, 0.8, 1\}$.

Point data $\left\{(z_i^{(k)}, \lambda_i^{(k)})\right\}_{i=1}^{5}$ of $n = 5$ reviewers regarding 4 articles are presented in Table (i is the reviewer's index, k is the article's index, $k = 1, \ldots, 4$).

	paper 1	paper 2	paper 3	paper 4
rev. 1	$(2, 0.8)$	$(2, 1)$	$(1, 0.8)$	$(0, 0.6)$
rev. 2	$(1, 1)$	$(2, 0.8)$	$(2, 0.6)$	$(1, 0.4)$
rev. 3	$(0, 0.8)$	$(0, 0.6)$	$(-1, 0.6)$	$(1, 1)$
rev. 4	$(3, 0.4)$	$(-1, 0.4)$	$(0, 0.6)$	$(2, 0.2)$
rev. 5	$(2, 0.6)$	$(1, 0.6)$	$(1, 1)$	$(2, 1)$

Let's transform each pair (z, λ) into a three-grade fuzzy estimate $\widetilde{x} = \widetilde{x}(z, \lambda) = (\mu_{\widetilde{x}}(L), \mu_{\widetilde{x}}(M), \mu_{\widetilde{x}}(H))$ using the following blur rule:

if $z \in H = \{1, 2, 3\}$ (high estimates), then $\widetilde{x} = \left(\frac{\lambda}{2z+\lambda}, \frac{\lambda}{z+\lambda}, \lambda\right)$;

if $z \in L = \{-3, -2, -1\}$ (low estimates), then $\widetilde{x} = \left(\lambda, \frac{\lambda}{|z|+\lambda}, \frac{\lambda}{2|z|+\lambda}\right)$;

if $z \in M = \{0\}$ (medium estimate), then $\widetilde{x} = \left(\frac{\lambda}{1+2\lambda}, \lambda, \frac{\lambda}{1+2\lambda}\right)$.

The fuzzy estimates obtained in this way are presented in Table.

	paper 1	paper 2	paper 3	paper 4
rev. 1	$\left(\frac{1}{6},\frac{2}{7},\frac{4}{5}\right)$	$\left(\frac{1}{5},\frac{1}{3},1\right)$	$\left(\frac{2}{7},\frac{4}{9},\frac{4}{5}\right)$	$\left(\frac{3}{11},\frac{3}{5},\frac{3}{11}\right)$
rev. 2	$\left(\frac{1}{3},\frac{1}{2},1\right)$	$\left(\frac{1}{6},\frac{2}{7},\frac{4}{5}\right)$	$\left(\frac{3}{23},\frac{3}{13},\frac{3}{5}\right)$	$\left(\frac{1}{6},\frac{2}{7},\frac{2}{5}\right)$
rev. 3	$\left(\frac{4}{13},\frac{4}{5},\frac{4}{13}\right)$	$\left(\frac{3}{11},\frac{3}{5},\frac{3}{11}\right)$	$\left(\frac{3}{5},\frac{3}{8},\frac{3}{13}\right)$	$\left(\frac{1}{3},\frac{1}{2},1\right)$
rev. 4	$\left(\frac{1}{16},\frac{2}{17},\frac{2}{5}\right)$	$\left(\frac{2}{5},\frac{2}{7},\frac{1}{6}\right)$	$\left(\frac{3}{11},\frac{3}{5},\frac{3}{11}\right)$	$\left(\frac{1}{21},\frac{1}{11},\frac{1}{5}\right)$
rev. 5	$\left(\frac{3}{23},\frac{3}{13},\frac{3}{5}\right)$	$\left(\frac{3}{13},\frac{3}{8},\frac{3}{5}\right)$	$\left(\frac{1}{3},\frac{1}{2},1\right)$	$\left(\frac{1}{5},\frac{1}{3},1\right)$
$\widetilde{v_L}^{(k)}$	$(1,m_3,0,0,0,0)$	$(1,m_3,0,0,0,0)$	$(1,m_3,0,0,0,0)$	$(1,m_3,0,0,0,0)$
$\widetilde{v_M}^{(k)}$	$(1,m_2,0,0,0,0)$	$(1,m_3,0,0,0,0)$	$(1,m_2,m_2,0,0,0)$	$(1,m_2,0,0,0,0)$
$\widetilde{v_H}^{(k)}$	$(0,0,1,m_2,0,0)$	$(0,0,1,0,0,0)$	$(0,0,1,0,0,0)$	$(0,0,1,0,0,0)$
$\mathbf{G}^{(k)}$	$\left(\frac{m_3}{1+m_3},\frac{m_2}{1+m_2},\frac{2+3m_2}{1+m_2}\right)$	$\left(\frac{m_3}{1+m_3},\frac{m_3}{1+m_3},2\right)$	$\left(\frac{m_3}{1+m_3},\frac{3m_2}{1+2m_2},2\right)$	$\left(\frac{m_3}{1+m_3},\frac{m_2}{1+m_2},2\right)$
$\mathbf{v}^{(k)}$	$(0,1,4)$	$(1,1,3)$	$(1,1,3)$	$(0,1,4)$

Using the method described above, we will find the FC of all reviewer ratings for all classes and for all articles.

We will obtain the following ranking of articles after **lexicographic** comparison of vectors $\mathbf{G}^{(k)}$, $k = 1, \ldots, 4$:

$$\varphi\left(\widetilde{\mathbf{x}}^{(2)}\right) > \varphi\left(\widetilde{\mathbf{x}}^{(4)}\right) > \varphi\left(\widetilde{\mathbf{x}}^{(1)}\right) > \varphi\left(\widetilde{\mathbf{x}}^{(3)}\right).$$

The same ranking will be obtained if the cardinalities of gradations are compared lexicographically.

If we take into account only the three-grade recommendations of reviewers ($L = \{-3, -2, -1\}$, $M = \{0\}$, $H = \{1, 2, 3\}$) and do not take into account the degree of confidence, we obtain the following vectors of cardinality of assessments $\mathbf{v}^{(k)} = \left(v_L(\mathbf{x}^{(k)}), v_M(\mathbf{x}^{(k)}), v_H(\mathbf{x}^{(k)})\right)$ for each k-article. Then the ranking of these articles will be as follows:

$$\varphi(\mathbf{x}^{(1)}) = \varphi(\mathbf{x}^{(4)}) > \varphi(\mathbf{x}^{(2)}) = \varphi(\mathbf{x}^{(3)}).$$

The main difference in the results of the new and non-fuzzy approaches is the rearrangement of alternatives $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$. The alternative $\mathbf{x}^{(1)}$ will be better than the $\mathbf{x}^{(2)}$ under non-blurred TA because the $\mathbf{x}^{(2)}$ has one low score, while the $\mathbf{x}^{(1)}$ has no low scores.

But a low score in the alternative $\mathbf{x}^{(2)}$ has a low degree of confidence. Therefore, it has little effect on the cardinality of low estimates under fuzzy TA.

Summary and Conclusion

- A new approach to TA and ranking of vector alternatives specified by fuzzy evaluations of criteria on a three-graded base set is proposed. This approach is based on calculating and comparing the FC of sets of estimates for each gradation for all criteria and for each alternative;
- The general properties that the FC of a set of fuzzy estimators must satisfy are discussed;
- The threshold procedure for constructing the FC of a set of fuzzy estimates is considered;
- The development of axiomatics for fuzzy threshold aggregation is one of the possible directions for future research.

References

- Aleskerov, F., Chistyakov, V., Kalyagin, V.: Social threshold aggregations. Social Choice Welfare **35**, 627–646 (2010)
- Aleskerov, F.T., Yakuba, V.I.: A Method for Threshold Aggregation of Three-Grade Rankings. Doklady Mathematics **75**(2), 322–324 (2007)
- Lepskiy, A.: Fuzzy Threshold Aggregation. In: Kahraman, C., Sari, I.U., Oztaysi, B., Cebi, S., Cevik Onar, S., Tolga, A.Ç. (eds.) Intelligent and Fuzzy Systems. INFUS 2023. LNNS, vol. 758, pp. 69–76. Springer, Cham (2023).

Thanks for you attention

alepskiy@hse.ru
https://www.hse.ru/en/org/persons/10586209