On the Conflict Measures Agreed with the Combining Rules

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Abstract. The conflict measures induced by the conjunctive and disjunctive combining rules are studied in this paper in the framework of evidence theory. The coherence of conflict measures with combining rules is introduced and studied. In addition, the structure of conjunctive and disjunctive conflict measures is studied in the paper. In particular, it is shown that the metric and entropy components can be distinguished in such measures. Moreover, these components are changed differently after combining of the bodies of evidence.

Keywords: conflict measure, evidence theory, combining rule

1 Introduction

Various factors must be considered when deciding about using of combining rules in the framework of evidence theory [3, 15]. The value of a conflict measure between bodies of evidence is the important characteristic when deciding about expediency of use a particular rule. In the recent years, the study of a conflict measures has been increasingly developing into an independent research area. Axiomatics and various approaches to the evaluation of the conflict between the bodies of evidence (external conflict) were considered in [1, 4, 8, 12, 13]. The notion of internal conflict of evidence studied in [2, 10, 11, 14]. But we are considering only the external conflict in this paper. The choice of a specific measure for estimation of a conflict depends on a solvable problem. For example, if we estimate conflict between bodies of evidence with the aim of decision making about combining of evidence, then the conflict measure must be agreed in some sense with the combining rule. So Dempster's rule of combination agrees naturally with a conjunctive conflict measure. The conditions of agreement for the other combining rules are not obvious. In the given paper we study the conflict measures that are induced by conjunctive and disjunctive combining rules. The link of consistency conditions with axioms of conflict measure is studied.

In addition, the structure of conjunctive and disjunctive conflict measures studied in this paper too. In particular, we showed that it is possible to allocate the metric and entropic components in such measures. Moreover, these components are changed in different ways when the bodies of evidence are aggregated.

The main aim of this paper consists in the study of some factors (the choice of a conflict measure, the consistency conditions with combining rules, the entropy of evidence, etc.) that should be considered when we make a decision on combining of bodies of evidence.

The structure of the remainder of the paper is as follows. First, in Section 2, we shall recall the basic concepts of evidence theory. Axioms of a conflict measure will be discussed in Section 3. The conflict measures that are induced by conjunctive and disjunctive combining rules are considered in Section 4. The notion of coherence of conflict measures and combining rules is introduced in Section 5. In Section 6, we showed that the metric and entropic components can be allocated in the conjunctive and disjunctive conflict measures. The change of metric and entropic components after combination bodies of evidence is discussed in Section 7. Finally, some conclusions are presented in Section 8.

2 Basic Definitions and Notations of Evidence Theory

We shall recall the basic concepts of evidence theory [3, 15]. Let X be a finite set and 2^X be a powerset of X. The mass function $m : 2^X \to [0,1]$ is considered and $\sum_{A\subseteq X} m(A) = 1$. The value $m(A)$ characterizes the relative part of evidence that the actual alternative from X belongs to set $A \in 2^X$. The subset $A \in 2^X$ is called a focal element, if $m(A) > 0$. Let $\mathcal{A} = \{A_i\}$ be a set of all focal elements of evidence. The pair $F = (\mathcal{A}, m)$ is called a body of evidence. Let $\mathcal{F}(X)$ be a set of all body of evidence on X.

If $\mathcal{A} = \{A\}$, then $F_A = (\mathcal{A}, m) = (A, 1)$ is called a categorical body of evidence. In particular F_X is called a vacuous body of evidence. If $F_j = (\mathcal{A}_j, m_j) \in \mathcal{F}(X)$, $0 \le \alpha_j \le 1$, $j = 1, ..., n$ and $\sum_{j=1}^n \alpha_j = 1$, then $F = (\mathcal{A}, m) \in \mathcal{F}(X)$, where $\mathcal{A} = \bigcup_{j=1}^n \mathcal{A}_j$, $m(A) = \sum_{j=1}^n \alpha_j m_j(A)$. In this case, we will write $F = \sum_{j=1}^n \alpha_j F_j$. In particular, any body of evidence $F = (\mathcal{A}, m)$ can be represented as $F = \sum_{A\in\mathcal{A}} m(A) F_A$.

Let we have two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$ which represent two information sources. The different combining rules R are considered in evidence theory: $R : \mathcal{F}(X) \times \mathcal{F}(X) \to \mathcal{F}(X)$. For example, the non-normalized conjunctive rule $D_0(F_1, F_2)$ is considered

$$m^{D_0}(A) = \sum\nolimits_{B\cap C=A} m_1(B) m_2(C), \quad A \in 2^X.$$

The value $K^D(F_1, F_2) = m^{D_0}(\emptyset)$ characterizes the amount of conflict between two sources of information (but not only, see [12]) described by the bodies of evidence F_1 and F_2. We call the value $K^D(F_1, F_2) = m^{D_0}(\emptyset)$ the conjunctive conflict measure. If $K^D \ne 1$, then the classical Dempster rule for combining of two evidence can be defined: $m^D(A) = \frac{1}{1-K^D} m^{D_0}(A)$, $A \ne \emptyset$, $m^D(\emptyset) = 0$. The conflict management in conjunctive rule was discussed in [9].

Dubois and Prade's disjunctive consensus rule is a dual rule to Dempster's rule in some sense. This rule is defined by a formula [6]:

$$m^{DP}(A) = \sum\nolimits_{B\cup C=A} m_1(B) m_2(C), \quad A \in 2^X.$$

In [7] a mixed conjunctive and disjunctive rule was discussed.

The negation (or complement) $\bar{F} = (\bar{\mathcal{A}}, \bar{m})$ of a body of evidence $F = (\mathcal{A}, m)$ is defined as $\bar{\mathcal{A}} = \{\bar{A} : \ A \in \mathcal{A}\}$ and $\bar{m}(A) = m(\bar{A})$ $\forall A \in \bar{\mathcal{A}}$, where $\bar{A}$ denotes the complement of A [5]. Note that if we have $F = \sum_{A \in \mathcal{A}} m(A) F_A$, then we have $\bar{F} = \sum_{A \in \mathcal{A}} m(A) F_{\bar{A}}$. The duality relation is true for the non-normalized conjunctive rule and disjunctive consensus rule by analogy with De Morgan's law [5]:

$$\overline{D_0(F_1, F_2)} = DP(\bar{F}_1, \bar{F}_2). \tag{1}$$

We shall consider also the dual body of evidence $F^{(-)} = (\bar{\mathcal{A}}, m^{(-)})$ with respect to body of evidence $F = (\mathcal{A}, m)$, where $m^{(-)}(\bar{A}) = \frac{1}{N-1}(1 - m(A))$ $\forall A \in \mathcal{A}$, $N = |\mathcal{A}| > 1$.

3 Axioms of Conflict Measures

In general, it is desirable that the conflict measure $K(F_1, F_2)$ between bodies of evidence satisfies the following conditions (axioms) [1, 4, 13]:

A1: $0 \le K(F_1, F_2) \le 1$ for all $F_1, F_2 \in \mathcal{F}(X)$ (non-negativity and normalization);

A2: $K(F_1, F_2) = K(F_2, F_1)$ for all $F_1, F_2 \in \mathcal{F}(X)$ (symmetry);

A3: $K(F, F) = 0$ for all $F \in \mathcal{F}(X)$ (nilpotency);

A4: $K(F', F) \ge K(F'', F)$, if $F' = (\mathcal{A}', m)$, $F'' = (\mathcal{A}'', m)$, where $\mathcal{A}' = \{A'_i\}$, $\mathcal{A}'' = \{A''_i\}$ and $A'_i \subseteq A''_i$ for all i and $F \in \mathcal{F}(X)$ (antimonotonicity with respect to imprecision of evidence);

A5: $K(F_X, F) = 0$ for all $F \in \mathcal{F}(X)$ (ignorance is bliss [4]);

A6: $K(F_A, F_B) = 1$, if $A \cap B = \emptyset$.

Furthermore, if we assume that the empty set can be a focal element (the value $m(\emptyset)$ can be interpreted as the degree of confidence in the fact that the true alternative $x \notin X$), then we assume that the axioms A3 and A5 are satisfied for all $F \in \mathcal{F}(X) \backslash \{F_\emptyset\}$ and we will also consider the following axiom:

A7: $K(F_\emptyset, F) = 1$ for all $F \in \mathcal{F}(X) \backslash \{F_\emptyset\}$.

The other axioms for conflict measures are also considered (see, e.g., [4]). We note that some axioms (for example, A4 and A6) are consistent with the conjunctive combining rule (see Section 5).

4 Conflict Measures Induced by Conjunctive and Disjunctive Combining Rules

Let us assume that the information from the two sources is described by means of two bodies evidence $F_1 = (\mathcal{A}_1, m_1)$ $F_2 = (\mathcal{A}_2, m_2)$. Then $K^D(F_1, F_2)$ can be considered as a conflict measure induced by conjunctive rule. This measure satisfies the axioms A1, A2, A4-A7.

Various conflict measures induced by the disjunctive consensus rule can be considered. These measures can satisfy certain axioms of the conflict measure. Below we consider the following conflict measures induced by a disjunctive rule (we will call them disjunctive conflict measures):

$$K_1^{DP}(F_1, F_2) = \sum\nolimits_{B \cup C = X} m_1(B) m_2(C), \quad K_2^{DP}(F_1, F_2) = 1 - K_1^{DP}(F_1, F_2).$$

Note that the measure $K_1^{DP}(F_1, F_2)$ satisfies only axioms A1, A2 and A7 (and the particular case of condition A6: $K(F_A, F_{\bar{A}}) = 1$). But the measure $K_2^{DP}(F_1, F_2)$ satisfies axioms A1, A2, A4, A5. The following relationship between conjunctive and disjunctive conflict measures is true. This relationship reflects the duality relation (1).

Proposition 1. $K_1^{DP}(F_1, F_2) = K^D(\bar{F}_1, \bar{F}_2)$.

The simple relations are true for the conjunctive and disjunctive conflict measures on the disjoint belief structure.

Proposition 2. *If* $F_1 = (\mathcal{A}, m_1)$ *and* $F_2 = (\mathcal{A}, m_2)$*, where* $A' \cap A'' = \emptyset \ \forall A', A'' \in \mathcal{A}$ $(A' \neq A'')$*, then: 1)* $K_1^{DP}(F_1, F_2^{(-)}) = \frac{1}{N-1} K^D(F_1, F_2)$*; 2)* $K_2^{DP}(F_1, \bar{F}_2) = K^D(F_1, F_2)$.

Proposition 3. *If* $F_1 = (\mathcal{A}_1, m_1)$*,* $F_2 = (\mathcal{A}_2, m_2)$ *and* $\mathcal{A}_1 = \bar{\mathcal{A}}_2$*, then* $K^D(F_1, F_2) = 1$ *implies* $K_1^{DP}(F_1, F_2) = 1$.

5 The Coherence of Conflict Measures and Combining Rules

The value of a conflict measure is an important factor for decision making about using of combining rules for aggregation of information from a few sources. In this case, the conflict measure serves as a priori characteristic of the applicability of the combining rule. The great value of a conflict measure means that we should not do the aggregation of these bodies of evidence. It is clear that the choice of a combining rule and a conflict measure must be coordinated to a certain degree in such problems. Let us consider the following matching conditions.

Definition 1. *A combining rule* R *and a conflict measure* K *are called positively agreed if* $K(F_1, F_2) \leq K(R(F_1, F_2), F_i)$*,* $i = 1, 2$ *for all* $F_1, F_2 \in \mathcal{F}(X)$*. The pair* R *and* K *is called negatively agreed if the opposite inequality holds.*

The positive (negative) coherence means that the value of a conflict measure between the resulting body of evidence and any operand will not decrease (will not increase) with respect to the value of a conflict measure between operands after application of combining rule.

Proposition 4. *1) A conjunctive (non-normalized) combining rule* D_0 *and a conflict measure* K^D *are positively agreed;*

2) a disjunctive combining rule DP *and a conflict measure* K_1^{DP} *are positively agreed;*

3) a disjunctive combining rule DP *and a conflict measure* K_2^{DP} *are negatively agreed.*

It is easy to see that the coherence of the conflict measure with the combining rule makes some axioms dependent or contradictory. For example,

1) if a conflict measure is positively agreed with a conjunctive combining rule or a disjunctive combining rule, then axiom A5 follows from axiom

A3 because $K(F_X,F) \leq K(D(F_X,F),F) = K(F,F) = 0$ and $K(F_X,F) \leq K(DP(F_X,F),F_X) = K(F_X,F_X) = 0$;

2) if a conflict measure is positively agreed with a conjunctive combining rule, then axiom A6 implies that $K(F_\emptyset,F_A) = 1$ for all $A \neq \emptyset$ (particular case of axiom A7) because $1 = K(F_A,F_{\bar{A}}) \leq K(D(F_A,F_{\bar{A}}),F_A) = K(F_\emptyset,F_A)$;

3) if a conflict measure is positively agreed with a disjunctive combining rule, then A5 and A6 axioms are contradictory as well as A3 and A7 axioms because $1 = K(F_A,F_{\bar{A}}) \leq K(DP(F_A,F_{\bar{A}}),F_A) = K(F_X,F_A) = 0$ and $1 = K(F_\emptyset,F) \leq K(DP(F_\emptyset,F),F) = K(F,F) = 0$;

4) if a conflict measure is negatively agreed with a conjunctive combining rule, then axiom A5 implies axiom A3 because $0 = K(F_X,F) \geq K(D(F_X,F),F) = K(F,F)$;

5) if a conflict measure is negatively agreed with a conjunctive combining rule, then axiom A7 implies that $K(F_A,F_{\bar{A}}) = 1$ for all $A \neq \emptyset$ (particular case of axiom A6) because $K(F_A,F_{\bar{A}}) \geq K(D(F_A,F_{\bar{A}}),F_A) = K(F_\emptyset,F_A) = 1$.

Thus, if we are talking about the desired conditions of conflict measure, then we must take into consideration the problem being solved, the used combining rule and, consequently, the type of their coherence.

6 Metric and Entropic Components of a Conflict Measure

When we take a decision on combining of bodies of evidence we pay attention not only on the value of a conflict measure. Consider the following example.

Example 1. Let us assume that there are three candidates $X = \{x_1,x_2,x_3\}$ for a certain position. Three experts expressed their preference to these candidates as three bodies of evidence $F_1 = \frac{1}{3}F_{\{x_1\}} + \frac{1}{3}F_{\{x_2\}} + \frac{1}{3}F_{\{x_3\}}$, $F_2 = \frac{1}{3}F_{\{x_1,x_2\}} + \frac{2}{3}F_{\{x_3\}}$, $F_3 = \frac{7}{8}F_{\{x_2\}} + \frac{1}{8}F_{\{x_2,x_3\}}$. The conjunctive conflict measures are equal $K^D(F_1,F_2) = 5/9$, $K^D(F_1,F_3) = 5/8$ and $K^D(F_2,F_3) = 7/12$, i.e. $K^D(F_1,F_2) < K^D(F_2,F_3) < K^D(F_1,F_3)$. We will choose for combining a couple of bodies of evidence F_1 and F_2 with the lowest measure of conflict. We get the new body of evidence after combining by Dempster's rule: $D(F_1,F_2) = \frac{1}{4}F_{\{x_1\}} + \frac{1}{4}F_{\{x_2\}} + \frac{1}{2}F_{\{x_3\}}$. The preference is given to a third candidate in this case. At the same time, the evidence F_1 is irrelevant because first expert did not give preference to any of the candidates. If we find a combination of the second and third bodies of evidence, then we get $D(F_2,F_3) = \frac{4}{5}F_{\{x_2\}} + \frac{1}{5}F_{\{x_3\}}$, i.e. the preference is given to a second candidate in this case. The situation is similar when we use a disjunctive conflict measure and a disjunctive rule: $K_1^{DP}(F_1,F_2) = \frac{1}{9} > K_1^{DP}(F_1,F_3) = K_1^{DP}(F_2,F_3) = \frac{1}{24}$; $DP(F_1,F_3) = \frac{7}{24}F_{\{x_2\}} + \frac{7}{24}F_{\{x_1,x_2\}} + \frac{9}{24}F_{\{x_2,x_3\}} + \frac{1}{24}F_{\{x_1,x_2,x_3\}}$; $DP(F_2,F_3) = \frac{7}{24}F_{\{x_1,x_2\}} + \frac{2}{3}F_{\{x_2,x_3\}} + \frac{1}{24}F_{\{x_1,x_2,x_3\}}$. We obtain approximately equal values of the mass function for three focal elements after combining the F_1 and F_3. On the contrary, the combination of the second and third sources gives us that the preferred candidate is in the pair $\{x_2,x_3\}$. This example can be explained by the fact that the first body of evidence has a uniform probability distribution. It has high Shannon entropy and it is better not to use for combining. However, the entropic and metric components can be isolated in the conflict measure.

Conjunctive conflict measure for two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$ can be rewritten as follows

$$K^D(F_1,F_2) = \sum_{B\in\mathcal{A}_1, C\in\mathcal{A}_2, B\cap C=\emptyset} m_1(B)m_2(C) = 1 - \sum_{B\cap C\neq\emptyset} m_1(B)m_2(C) =$$

$$= \tfrac{1}{2}\left(2 - 2\sum_{B,C} q_{B,C} m_1(B)m_2(C)\right) - \sum_{B,C}\left(t_{B,C} - q_{B,C}\right) m_1(B)m_2(C), \qquad (2)$$

where $Q = (q_{B,C})$ is a symmetric positive definite matrix which satisfies the conditions: 1) $q_{B,C} \in [0,1]\ \forall B,C \in 2^X$; 2) $q_{B,C} = 0$, if $B\cap C = \emptyset$; 3) $q_{B,B} = 1\ \forall B \in 2^X$; $T = (t_{B,C})$, $t_{B,C} = \begin{cases} 1, & B\cap C \neq \emptyset, \\ 0, & B\cap C = \emptyset. \end{cases}$ Let $R = (r_{B,C})$, $r_{B,C} = t_{B,C} - q_{B,C}$. For example, Jaccard index, $q_{B,C} = \frac{|B\cap C|}{|B\cup C|}$, $\forall B, C \neq \emptyset$ is an example of coefficients $q_{B,C}$. We will consider a scalar product $(\mathbf{x},\mathbf{y})_Q := \mathbf{x}^T Q\mathbf{y}$ and corresponding norm $\|\mathbf{x}\|_Q := \sqrt{\mathbf{x}^T Q\mathbf{x}}$ in the real vector space $A_{2^{|X|}-1}$. In particular, if $q_{B,C}$ is Jaccard index, then $d_J(F_1,F_2) = \frac{1}{\sqrt{2}}\|\mathbf{m}_1 - \mathbf{m}_2\|_Q$ is Jousselme distance [8] that is widely used in evidence theory.

Let $S_X = \left\{\mathbf{t} = (t_k)_{k=1}^{2^{|X|}-1} :\ t_k \in [0,1]\ \forall k,\ \sum_k t_k = 1\right\}$ is a simplex. We consider a functional $E_Q : S_X \to [0,1]$,

$$E_Q(F) = E_Q(\mathbf{m}) = 1 - \|\mathbf{m}\|_Q^2 = \sum_B m(B)\left(1 - \sum_C q_{B,C} m(C)\right), \quad F = (\mathcal{A}, m).$$

This functional is close to an entropy functional in some of its properties: $\mathbf{t}^{(\max)} = \arg\max_{S_X} E_Q(\mathbf{t}) = \arg\min_{S_X} \|\mathbf{t}\|_Q^2$, $\mathbf{t}^{(\min)} = \arg\min E_Q(\mathbf{t})$, if $\exists j : t_j^{(\min)} = 1$ and $t_k^{(\min)} = 0\ \forall k \neq j$ (categorical evidence), $E_Q(\mathbf{t}^{(\min)}) = 0$. The next Proposition follows from (2).

Proposition 5. *We have for the conjunctive conflict measure*

$$K^D(F_1,F_2) = \tfrac{1}{2}\left(E_Q(\mathbf{m}_1) + E_Q(\mathbf{m}_2) + \|\mathbf{m}_1 - \mathbf{m}_2\|_Q^2\right) - \sum_{B,C} r_{B,C} m_1(B)m_2(C). \quad (3)$$

The formula (3) shows that the conjunctive conflict measure can be represented as a sum of average value of entropy-type functionals of bodies of evidence, the square distance between bodies of evidence and a last summand that characterizes the interaction of weakly intersecting focal elements.

Corollary 1. *If $F_1 = (\mathcal{A}, m_1)$ and $F_2 = (\mathcal{A}, m_2)$, where $A' \cap A'' = \emptyset\ \forall A', A'' \in \mathcal{A}$, then*

$$K^D(F_1,F_2) = \tfrac{1}{2}\left(E_I(\mathbf{m}_1) + E_I(\mathbf{m}_2)\right) + \tfrac{1}{2}\|\mathbf{m}_1 - \mathbf{m}_2\|_I^2, \qquad (4)$$

where I is the identity matrix and $\|\mathbf{x}\|_I := \sqrt{\mathbf{x}^T\mathbf{x}}$ is the Euclidean norm.

Note that functional $E_I(\mathbf{t}) = \sum_B t(B)(1 - t(B))$ is defined on the simplex S_X and satisfies the conditions: $\mathbf{t}^{(\max)} = \arg\max E_I(\mathbf{t})$, if $t_k^{(\max)} = \frac{1}{2^{|X|}-1}\ \forall k$ (uniform distribution), $E_I(\mathbf{t}^{(\max)}) = 1 - \frac{1}{2^{|X|}-1}$; $\mathbf{t}^{(\min)} = \arg\min E_I(\mathbf{t})$, if $\exists j :$

$t_j^{(\min)} = 1$ and $t_k^{(\min)} = 0$ $\forall k \neq j$ (categorical evidence), $E_I(\mathbf{t}^{(\min)}) = 0$. In addition, we have $E_I(\mathbf{t}) \leq S(\mathbf{t}) := -\sum_k t_k \log_2 t_k$ (Shannon entropy). Thus, the conjunctive conflict measure is equal in this case the average value of the entropy-type functionals and the square of the distance between the mass functions of the two bodies of evidence.

Note that the conjunctive conflict measure satisfies the triangle inequality on the disjoint belief structures.

Proposition 6. *If $F_i = (\mathcal{A}, m_i)$, $i = 1, 2, 3$, where $A' \cap A'' = \emptyset$ $\forall A', A'' \in \mathcal{A}$, then $K^D(F_1, F_3) \leq K^D(F_1, F_2) + K^D(F_2, F_3)$.*

Proposition 2 implies that we have the following representation for a disjunctive conflict measure and a special case of belief structures $F_1 = (\mathcal{A}, m_1)$ and $F_2 = (\mathcal{A}, m_2)$, where $A' \cap A'' = \emptyset$ $\forall A', A'' \in \mathcal{A}$, $N = |\mathcal{A}| > 1$:

$$K_1^{DP}(F_1, F_2^{(-)}) = \frac{1}{2(N-1)} \left(E_I(\mathbf{m}_1) + E_I(\mathbf{m}_2) \right) + \frac{1}{2(N-1)} \left\| \mathbf{m}_1 - \mathbf{m}_2 \right\|_I^2 ,$$

$$K_2^{DP}(F_1, \bar{F}_2) = K^D(F_1, F_2) = \frac{1}{2} \left(E_I(\mathbf{m}_1) + E_I(\mathbf{m}_2) \right) + \frac{1}{2} \left\| \mathbf{m}_1 - \mathbf{m}_2 \right\|_I^2 .$$

7 Changing of Metric and Entropic Components of a Conflict Measure after Combining

By definition, we have that a conflict measure is not decreased after combining of bodies of evidence in the case of positive compatibility. On the other hand, metric and entropic components can be isolated in the conjunctive conflict measure. We have the question about changing of these parts when bodies of evidence are combined.

Proposition 7. *If $F_1 = (\mathcal{A}, m_1)$ and $F_2 = (\mathcal{A}, m_2)$, where $\emptyset \notin \mathcal{A}$ and $A' \cap A'' = \emptyset$ $\forall A', A'' \in \mathcal{A}$, then the metric component of a conjunctive conflict measure does not decrease after application of a conjunctive rule.*

The entropic component of a conjunctive conflict measure can be increased or decreased after application of a conjunctive rule.

Proposition 8. *If $F_1 = (\mathcal{A}, m_1)$ and $F_2 = (\mathcal{A}, m_2)$, where $\emptyset \notin \mathcal{A}$ and $A' \cap A'' = \emptyset$ $\forall A', A'' \in \mathcal{A}$, then $E_I(DP(F_1, \bar{F}_2)) \geq E_I(\bar{F}_2)$.*

By other words, the value of entropy-type functional does not decrease after combining of bodies of evidence F_1 and $\bar{F}_2$ with the help of disjunctive rule with respect to value of entropy-type functional of body $\bar{F}_2$.

The metric component of a disjunctive conflict measure can be increased or decreased after application of a disjunctive rule.

8 Conclusions

Conflict measures induced by the conjunctive and disjunctive combining rules were studied in this paper. In particular, some of the consistency conditions between the combining rules and conflict measures were discussed. The relationship of consistency conditions and the axiomatic of a conflict measure is shown.

In addition, it is shown that the metric and entropic components can be isolated into the conjunctive conflict measures. It is shown that the entropic component of evidence is an important characteristic (together with the value of a conflict measure) in decisions about the choice of the bodies of evidence for combining. It is shown in some special cases (disjoint belief structures) that the metric component of conjunctive conflict measure is not decreased after applying of conjunctive combining rule. In addition, it is shown that the value of entropic component is not decreased after combining of bodies of evidence with the help of disjunctive rules.

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