Aggregation of Forecasts and Recommendations of Financial Analysts in the Framework of Evidence Theory

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Abstract. The article is dedicated to the method of aggregation of financial analysts' recommendations in the framework of the evidence theory. This method considered on the example of Russian stock market and the quality of the obtained results was compared with the classical consensus forecast. It is shown that the combination rules, which are widely developed in the theory of evidence, allow aggregating the recommendations of analysts taking into account the historical reliability of information sources, the nature of the taken decisions (pessimism-optimism), the conflict between forecasts and recommendations, etc. In most cases it turned out that, obtained aggregated forecasts are more accurate than consensus forecast.

Keywords: evidence theory, combining rule, recommendations of financial analysts, consensus forecast, discounting of evidence

1 Introduction

The information plays the key role when investors make decisions in the stock market. Forecasts and recommendations of financial analysts stand out among the various information factors.

Although, the scientific literature evaluates the importance of analysts' recommendations on certain aspects of investor's decision-making in different ways, most researchers tend to believe that they are useful for decision-making. So in [13] (and earlier works) it was shown that shifts in the recommendations of analysts has a 20% impact on the change in share prices. Some researchers, for example, [7] argue (and show this) that analysts' recommendations better predict EPS values than time series models (for example, GARCH models). The relation between the accuracy of forecasts and the quality of financial institutions was investigated in [21]. The influence of conflict of interest on the quality of forecasts was considered in [11]. A mathematical model that connects the accuracy of analyst predictions and the relevance of recommendations was constructed and investigated in [20]. Factors affecting the accuracy of the analysts' forecasts have been studied in [8]. The impact of forecasts on the change in the value of shares was examined in [2]. In [12] it was shown that the recommendations for the purchase of shares are a significant positive signal to the growth of profitability, and the recommendations of the sale - a significant negative signal. At the same time, it was demonstrated that the effectiveness of recommendations is not uniform across companies and industries. In particular, the effectiveness of recommendations is not high for companies with a low level of transparency.

In [15] the information content and the relationship of analysts' recommendations with insider information were investigated. In [4] were analyzed the changes in the nature of the recommendations of analysts from investment banks with different reputations during the IPO period. A large review of foreign scientific literature on the analysis of recommendations of financial analysts up to 2009 can be found in [22].

Among the tasks associated with evaluating the recommendations of financial analysts, the important challenge is to aggregate recommendations and forecasts. So, in [3] it was demonstrated that taking into account the recommendations aggregated at the country level improves profitability on the international stock market. In [14] it is shown that the inclusion of changes in aggregated recommendations on average positively affects for income and profit. In [16] was showed the importance of the recommendations combining and forecasts of the target price for building a profitable investment strategy.

A weighted consensus forecast is usually used as a general method of aggregation (for example, in [3], the values of market capitalization of firms are appointed as weights).

At the same time, in a number of studies it was shown that aggregate forecasts should be treated with caution. For example, in [19] it was argued that the average value is inefficient for aggregating forecasts. This inefficiency increases with the growth in the number of forecasts in aggregation. Also there were offered some procedures for selecting "good" forecasts for aggregation.

Therefore, the actual problem is to choose such aggregation procedures, which most fully take into account information from individual analysts, the history of their forecasts, their correlation (conflict), etc. All the noted features (uncertainty, reliability, conflicts) can be described within the theory of evidence (Dempster-Shafer theory, theory of confidence functions [9, 24]).

Recently, the theory of evidence is widely used in financial and economic analysis, for example, in forecasting of investments' profitability on the basis of interval expert assessments [25], in marketing analysis of data [17], in forecasting of income on the stock market [1], in foresight studies [26], etc. This work is an attempt to analyze the aggregation of analysts' recommendations in the framework of the evidence theory on the example of the Russian stock market.

The rest of this paper is organized as follows. In Section 2, we present the background of evidence theory including combining rules. The survey of financial analysts' data on the Russian Stock Market is given in Section 3. The construction of bodies of evidence of financial analysts' and implementation of different aggregation procedure are described in Section 4. Experimental results are presented in Section 5. The last section summarizes the paper.

2 Background of Evidence Theory

Let Ω be a some universal set (frame of discernment) of all possibility values of experimental results (observations, alternatives, etc.), $\mathcal{P}(\Omega)$ be a powerset of Ω. A mass function (also called basic belief assignment or basic probability assignment) is a set function $m : \mathcal{P}(\Omega) \to [0,1]$ that satisfies the condition $\sum_{A\in\mathcal{P}(\Omega)} m(A) = 1$. In particular, mass function can be considered as a relative frequency that the true alternative belongs to the set A [9], i.e. $m(A) = c/N$, where c is the number of observed sets $A \subseteq \Omega$, N is the total number of observations.

The subset $A \subseteq \Omega$ is called a focal element if $m(A) > 0$. The pair $F = (\mathcal{A}, m)$ from the set of all focal elements $\mathcal{A} = \{A\}$ and corresponding mass function $m(A)$, $A \in \mathcal{A}$ is called a body of evidence. Let $\mathcal{F}(\Omega)$ be a set of all possible bodies of evidence on Ω. The body of evidence is said to be categorical (and this is denoted as $F_A = (A, 1)$) if it has only one focal set. In particular, the body of evidence $F_\Omega = (\Omega, 1)$ is said to be vacuous, since it does not carry any information about belonging of true alternative to any subset of Ω. If $F_j = (\mathcal{A}_j, m_j) \in \mathcal{F}(\Omega)$, $j = 1, ..., n$ and $\sum_{j=1}^{n} \alpha_j = 1$, $0 \le \alpha_j \le 1$, $j = 1, ..., n$, then $F = (\mathcal{A}, m) \in \mathcal{F}(\Omega)$, where $\mathcal{A} = \bigcup_{j=1}^{n} \mathcal{A}_j$, $m(A) = \sum_{j=1}^{n} \alpha_j m_j(A)$. In this case, we will write $F = \sum_{j=1}^{n} \alpha_j F_j$. In particular, any body of evidence $F = (\mathcal{A}, m)$ can be represented as $F = \sum_{A\in\mathcal{A}} m(A) F_A$.

The body of evidence is said to be simple if it has at most two focal sets and, if it has two, Ω is one of those. The simple body of evidence can be represented in the form $F_A^\omega = (1-\omega)F_A + \omega F_\Omega$, where $\omega \in [0,1]$. In particular, $F_A^0 = F_A$ and $F_A^1 = F_\Omega$.

The possibility of combining of bodies of evidence is the one of the convenient tools in the evidence theory. This corresponds to the aggregation of information getting from different sources. There are a few combining rules (see review, for example, in [23]). Historically, the Dempster's rule [9] was the first among combining rules. Suppose there are two independent groups of experts who provide their forecasts, given in the form of two bodies of evidence $F_1 = (\mathcal{A}_1, m_1)$ and $F_2 = (\mathcal{A}_2, m_2)$ on the same universal set Ω. Then the mass function $m = m_1 \oplus m_2$ of new body of evidence obtained with the help of Dempster's rule is calculated by the formula

$$m(A) = \frac{1}{1-K} \sum_{\substack{B\cap C=A,\\ B\in\mathcal{A}_1, C\in\mathcal{A}_2}} m_1(A) m_2(B), \quad A \ne \emptyset,$$

where $K = K(F_1, F_2) = m(\emptyset) = \sum_{B\cap C=\emptyset} m_1(A) m_2(B)$. The value $K \in [0,1]$ characterizes the degree of conflictness of information sources. The large value of this parameter means that the sources provide conflicting information. In particular, if information sources are absolutely conflict (i.e. $B \cap C = \emptyset$ for all $C \in \mathcal{A}_1$, $C \in \mathcal{A}_2$) then the Dempster's rule is not applicable because in this case $K = 1$.

Since the combining rule $\oplus$ is an associative operation, then any finite number of bodies of evidence can be combined.

The Dempster's rule is an optimistic rule in the following sense. If two sources of information states that true alternative belongs to set A and B correspondingly, then new evidence obtained with the help of Dempster's rule will state that true alternative belongs to set $A \cap B$. By other words we get categorical evidence $F_{A\cap B}$ after combining of non conflict categorical evidence F_A and F_B ($A \cap B \neq \emptyset$). Notice that if we combined simple evidence $F_A^{\omega_A}$ and $F_B^{\omega_B}$, then we get evidence with three focal elements A, B, Ω in the case $A \cap B = \emptyset$ or with four focal elements $A, B, A \cap B, \Omega$ in the otherwise.

Example 1. Let one expert predicts that the price of shares of a some company through a month will be in the interval [40,50) with the value of mass function $m_1([40, 50)) = 0.7$ or in the interval [50,55) with $m_1([50, 55)) = 0.3$. Other expert predicts that the price of shares of same company through a month will be in the interval [40,48) with the value of mass function $m_2([40, 48)) = 0.6$ or in the interval [48,52) with $m_2([48, 52)) = 0.4$. Then the conflict between these bodies of evidence is equal $K = 0.3 \cdot 0.6 = 0.18$. We can get new body of evidence with the help of Dempster's rule with mass function $m = m_1 \oplus m_2$: $m([40, 48)) = \frac{1}{1-K} \cdot 0.7 \cdot 0.6 = \frac{21}{41}$, $m([48, 50)) = \frac{1}{1-K} \cdot 0.7 \cdot 0.4 = \frac{14}{41}$, $m([50, 52)) = \frac{1}{1-K} \cdot 0.3 \cdot 0.4 = \frac{6}{41}$.

Let all focal elements of body of evidence $F = (\mathcal{A}, m)$ are bounded sets in $\mathbb{R}$. Then the lower and upper boundaries of expectation of belonging of true alternative can be calculated:

$$\underline{\mathrm{E}}[F] = \sum\nolimits_{A\in\mathcal{A}} m(A) \inf\{A\}, \quad \overline{\mathrm{E}}[F] = \sum\nolimits_{A\in\mathcal{A}} m(A) \sup\{A\}.$$

For example, we have for data from Example 1 for combining evidence $\underline{\mathrm{E}}[F] = \frac{21}{41} \cdot 40 + \frac{14}{41} \cdot 48 + \frac{6}{41} \cdot 50 \approx 44,19$ and $\overline{\mathrm{E}}[F] = \frac{21}{41} \cdot 48 + \frac{14}{41} \cdot 50 + \frac{6}{41} \cdot 52 \approx 49,27$.

If the information sources are not reliable, then this can be taken into account with the help of discounting operation that was proposed in [24]. The degree of reliability of information source estimated with the help of discount coefficient $\alpha \in [0, 1]$. In this case values of mass function are recalculated by the formula:

$$m^\alpha(A) = (1 - \alpha)m(A) \quad \forall A \neq \Omega, \quad m^\alpha(\Omega) = \alpha + (1 - \alpha)m(\Omega).$$

If $\alpha = 1$, then it means that information source is absolutely not reliable. If $\alpha = 0$, then it means that information source is absolutely reliable. The some combining rule (for example, Dempster's rule) applied after discounting of initial bodies of evidence. Notice that the categorical evidence F_A is transformed to simple evidence F_A^α after discounting, but the simple evidence F_A^ω is transformed to simple evidence $F_A^{\alpha+(1-\alpha)\omega}$ after discounting.

Example 2. If in the Example 1 the first information source has reliability $\alpha_1 = 0.2$, but a second information source has absolutely reliability (i.e. $\alpha_2 = 0$), then we will obtain a new evidence after discounting of the first evidence (providing that $\Omega = [0, 100]$): $m_1([40, 50)) = 0.8 \cdot 0.7 = 0.56$, $m_1([50, 55)) = 0.8 \cdot 0.3 = 0.24$, $m_1([0, 100]) = 0.2$. Then we will obtain a new evidence after combining with the help of Dempster's rule the discounted first evidence with the second evidence: $K = 0.24 \cdot 0.6 = 0.144$, $m([40, 48]) = \frac{1}{1-K} \cdot (0.56 \cdot 0.6 + 0.2 \cdot 0.6) = \frac{57}{107}$, $m([48, 50]) = \frac{1}{1-K} \cdot 0.56 \cdot 0.4 = \frac{28}{107}$, $m([50, 52]) = \frac{1}{1-K} \cdot 0.24 \cdot 0.4 = \frac{12}{107}$, $m([48, 52]) = \frac{1}{1-K} \cdot 0.2 \cdot 0.4 = \frac{10}{107}$. In this case the lower and upper boundaries of expectation of belonging of price of shares will be equal $\underline{\mathrm{E}}[F] = \frac{57}{107} \cdot 40 + \frac{28+10}{107} \cdot 48 + \frac{12}{107} \cdot 50 \approx 43,96$ and $\overline{\mathrm{E}}[F] = \frac{57+10}{107} \cdot 48 + \frac{28}{107} \cdot 50 + \frac{12}{107} \cdot 52 \approx 48,97$. Boundaries will decrease a little. This corresponds to the more reliable second evidence.

In addition, we have $K \neq 1$ for discounted bodies of evidence with non zero coefficients. Therefore the Dempster's rule is applicable in this case. We will use only the Dempster's rule in this paper for combining of evidence.

3 The Survey Data

As a rule, an expert evaluation of the share price behavior on a limited time horizon consists of two indicators: the target price and direct recommendation. Forecasts of experts can be characterized by the time of the forecast actuality. There are allocated the most common time horizons: weekly, monthly, quarterly and annual. The target price is the share price expected by the expert at the end of the forecast period.

Recommendations of analysts can take the values "sell", "hold", "buy", which correspond to different expected growth rates of share prices: "sell" forecast corresponds to the low or negative expected potential growth of the security price; "hold" corresponds to the average expected potential growth of the security price; "buy" corresponds to the high expected potential growth of the security price. Some banks use other designations, which, however, can be compared with the scale of "sell-hold-buy".

Seven Russian banks and three analytical companies that provide their annual forecasts for blue chips (only common shares) represented on the Russian stock market during January 2010-May 2016 act as sources of expert assessments in this study.

There is the list of the information sources (banks) under consideration: Gazprombank (GPB), Otkritie (BO), Raiffeisen Bank (RB), Renaissance Credit (RK), Sberbank of Russia (SB), Uralsib (URS), VTB (BVTB).

The list of considered analytical companies: AK BARS Finance (ABF), BCS, Finance Investment Company (UFIC).

The list of companies and their stock designations that were considered: Gazprom (GAZP), Lukoil (LKOH), Rosneft (ROSN), Sberbank of Russia (SBER), Magnit (MAGN), Surgutneftegas (SNGSP), Nornickel (GMKN), VTB (VTBR), Transneft (TRNFP), Tatneft (TATN), Mobile TeleSystems (MTSS), Severstal (CHMF), ALROSA (ALRS), Novatek (NVTK), Aeroflot (AFLT), Uralkali (URKA).

Also, using the RBC QUOTE and Bloomberg Terminal, the data on the real value of the shares of these companies in the period from January 2010 to May 2016 were collected.

4 Implementation

4.1 Determination of Focal Elements and Bodies of Analysts' Evidence

In order to apply the methods of the evidence theory to solve the problem of aggregating the financial analysts' forecasts, it is necessary to move from point expert assessments to interval estimates. For this purpose, we established compliance between the predicted growth rate of shares ("sell-hold-buy") and the corresponding interval, unique and rather stable for each bank and the analytical company. This compliance is implicitly set by the technique of creation of the forecast, chosen measure of the wrong decision risk, a macroeconomic background, etc.

As the characteristic of growth rates of the share price, we used the relative target price, which represents the relation of the target price $Tgt(stock, t)$ of the share *stock* to the actual price of this share $Pcl(stock, t)$ on the date of the forecast submission t: $Crv(stock, t) = \frac{Tgt(stock,t)}{Pcl(stock,t)}$.

Then, for each data source$i$the boundary values for intervals of a relative price were determined. Obtained intervals correspond to recommendations of "sell-hold-buy" scale. This boundary values were calculated under the error (when observed forecast for the relative price does not match the recommendation for constructed interval, which is hit by this relative price) minimization condition. Besides, were determined the lower bound for "sell" interval as the minimum value of the relative price and the upper bound of the interval "buy" as the maximum value of the relative price. For example, for Sberbank of Russia (SBER) a focal interval of a relative price, which corresponds to recommendation "sell", equal to $S_{SBER} = [0.36, 0.61)$, for recommendation "hold" - $H_{SBER} = [0.61, 1.17)$, for recommendation "buy" - $B_{SBER} = [1.17, 1.69)$. In this case, the percentage of errors (forecasts of relative stock prices hits "not their" intervals of recommendations) for Sberbank of Russia is 8.57.

Thus, taking into account the fact that during one year one analytical company gives several recommendations, for each analytical company i and each *stock* body of evidence can be constructed $F_{i,stock} = (\mathcal{A}_{i,stock}, m_{i,stock})$, $stock = \{\text{GAZP}, \ldots, \text{URKA}\}$, $i = \{\text{ABF}, \ldots, \text{BVTB}\}$. Each body of evidence has not more than three focal elements (without discounting) $S_{i,stock}$, $H_{i,stock}$, $B_{i,stock}$, and mass functions $m_{i,stock}(A)$ equal to relative frequency of recommendation that was submitted by the source i and hit one of constructed intervals of expected relative stock price: $A \in \{S_{i,stock}, H_{i,stock}, B_{i,stock}\}$. The set Ω is added to the set focal elements in the case of discounting.

We can formulate the problem of finding the optimal body of evidence with account of revision of forecasts. Let we have n categorical bodies of evidence that ordered by the time F_{A_s}, where $A_s \in \{S_{i,stock}, H_{i,stock}, B_{i,stock}\}$, $s = 1, \ldots, n$. Here F_{A_s} is a recommendation of i-th source in during a year. We will consider a combining evidence $F(\alpha_1, \ldots, \alpha_n) = \oplus_{s=1}^{n} F_{A_s}^{\alpha_s}$ for finding of recommendation of i-th source on the end a year with account of revision of forecasts.

The $F(\alpha_1, \ldots, \alpha_n)$ is a combining evidence obtained with the help of Dempster's rule with discounting and coefficients $1 \geq \alpha_1 \geq \ldots \geq \alpha_n \geq 0$. Here $F_{A_s}^{\alpha_s} = (1-\alpha_s)F_A + \alpha_s F_\Omega$, where $A \in \{S_{i,stock}, H_{i,stock}, B_{i,stock}\}$. The inequality $1 \geq \alpha_1 \geq \ldots \geq \alpha_n \geq 0$ corresponds a requirement that last recommendations are more important for combining.

We will consider the following criteria for optimization $C(\alpha_1, \ldots, \alpha_n) = (\mathrm{E}_0[F(\alpha_1, \ldots, \alpha_n)] - p)^2$, where p is an actual last "pre-forecast" relative price of the share, $\mathrm{E}_0[F] = \frac{1}{2}(\underline{\mathrm{E}}[F] + \overline{\mathrm{E}}[F])$ is the middle value of the interval of expectation of the forecast price. It is required to find such coefficients $\alpha_1, \ldots, \alpha_n$, that satisfies the condition $1 \geq \alpha_1 \geq \ldots \geq \alpha_n \geq 0$ and $C(\alpha_1, \ldots, \alpha_n) \to \min$.

Example 3. Let $n = 4$, $F_1 = F_2 = F_4 = F_S$ ("sell") and $F_3 = F_H$ ("hold"). Then

$$F(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = F_S^{\alpha_1} \oplus F_S^{\alpha_2} \oplus F_H^{\alpha_3} \oplus F_S^{\alpha_4} = m(S)F_S + m(H)F_H + m(\Omega)F_\Omega.$$

The conflict of discounting bodies of evidence is equal $K = K\left(F_S^{\alpha_1}, F_S^{\alpha_2}, F_H^{\alpha_3}, F_S^{\alpha_4}\right) = (1-\alpha_3)(1-\alpha_1\alpha_2\alpha_4)$. The values of a mass function after combining of discounting bodies of evidence are equal:

$$m(S) = \frac{\alpha_3(1-\alpha_1\alpha_2\alpha_4)}{1-K}, \quad m(H) = \frac{\alpha_1\alpha_2(1-\alpha_3)\alpha_4}{1-K}, \quad m(\Omega) = \frac{\alpha_1\alpha_2\alpha_3\alpha_4}{1-K}.$$

Consequently, we have

$$C(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (\mathrm{E}_0[F(\alpha_1, \alpha_2, \alpha_3, \alpha_4)] - p)^2 =$$

$$\left(\frac{\alpha_3(1-\alpha_1\alpha_2\alpha_4)S_0 + \alpha_1\alpha_2(1-\alpha_3)\alpha_4 H_0 + \alpha_1\alpha_2\alpha_3\alpha_4\Omega_0}{\alpha_3 + \alpha_1\alpha_2\alpha_4 - \alpha_1\alpha_2\alpha_3\alpha_4} - p\right)^2,$$

where S_0, H_0, Ω_0 are middles of intervals of relative prices of S, H and Ω correspondingly. We will solve a problem $C(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \to \min$ subject to the condition $1 \geq \alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4 \geq 0$ and we will find the optimal evidence F. For example, if $S_0 = 0.7$, $H_0 = 1.1$, $\Omega_0 = 0.9$ and $p = 0.8$, then we obtain optimal coefficients $\alpha_1 = \alpha_2 = 1$, $\alpha_3 \approx 0.34$, $\alpha_4 \approx 0.13$ and $F \approx 0.7F_S + 0.2F_H + 0.1F_\Omega$.

Note that, if we combines I bodies of evidence of the form $m(S)F_S + m(H)F_H + m(B)F_B + m(\Omega)F_\Omega$, then we will obtain a new evidence, in which there can be up to 4^I of the focal elements.

4.2 Different Combining Strategies and Forecast Error Estimation

Further, for each stock and each year the new (aggregated) body of evidence was calculated by means of Dempster's rule. Also, the upper and the lower edges of expected value of relative price were constructed. It should be mentioned that, forecasts of analysts can be completely conflict (i.e. $K(F_1, F_2) = 1$) or have a high degree of conflict. In the first case the Dempster's rule is not applicable. In addition, in [18] it was shown that investors should be cautious about those recommendations that have high degree of conflict (in particular, they have significant differences in forecasted target price). On the other hand, combining only non-conflicting recommendations does not contribute to the consideration of the opinions of experts who have an alternative point of view. Therefore, it is necessary to define the rules according to which the sources for combining will be selected. In this paper, two alternative rules for selecting sources were considered.

1. All sources were ranked by the increase in the degree of conflict and the combination of evidence began with a pair of the least conflicting sources. The algorithm of ranking is:

- to choose the pair of evidences $(F_i, F_j) = \underset{F' \neq F''}{\arg\min} K(F', F'')$;
- to choose three evidences $(F_i, F_j, F_s) = \underset{F \neq F_i, F \neq F_j}{\arg\min} K(F_i, F_j, F)$ (including previous pair from the first step) and so on. In this case $K(F_{i_1}, ..., F_{i_s}) = \sum_{A_{i_1} \cap ... \cap A_{i_s} = \emptyset} m_{i_1}(A_{i_1})...m_{i_s}(A_{i_s})$ is conflict for s evidences;
- the algorithm stops when $K(F_i, F_j, ..., F_s) > K_0$. Assume, that $K_0 = 0.95$.

2. For each data source i the degree of forecast's reliance was evaluated for each $stock = \{\text{GAZP}, \ldots, \text{URKA}\}$ basing on data of a previous period

$$\delta_{i,stock} = \frac{1}{N} \sum_t \frac{|Crv_{real}(stock, t) - Crv_{forecast}(stock, t)|}{\max\{Crv_{real}(stock, t), Crv_{forecast}(stock, t)\}},$$

where N is the number of forecasts of the share price $stock$ during the period, $Crv_{real}(stock, t)$ is the actual relative price of the share $stock$ at the moment t, $Crv_{forecast}(stock, t)$ is the forecasted relative price of the share $stock$ at the moment t. All sources were ranked by the increase in degree of forecast's reliance and the combination of evidence began with a pair of the most reliable sources. If the data from the first two sources of the ranked row were in conflict, then the first source from the row was selected. Further, the following in turn among sources, which did not in the conflict with already combined, was chosen.

The forecast history of the data sources considered by means of discounting. The value $\delta_{i,stock} \in [0, 1]$ was chosen as a coefficient of discounting. In case when in current period one of data sources represents forecasts of the relative price for the stock for the first time (this source have not submitted forecasts for this share in previous periods), it is possible to consider several approaches to such forecasts. The first scenario: it is possible to treat recommendations of shares, rather new to this source, with high degree of trust and to choose for them small value of discounting coefficient, for example, $\delta_{i,stock} = 0.1$. The second, neutral, the scenario assumes smaller trust to recommendations for shares, rather new to this source, $\delta_{i,stock} = 0.5$. The third scenario assumes $\delta_{i,stock} = 0.75$, i.e. degree of reliability of new sources is supposed a little higher then degree reliability of sources, which forecasts in the last period were very far from real values.

Rather "old" sources of information we will keep to two strategies. In the first case – "with censorship" – for a combination there were chosen only those sources, which coefficient of discounting is less than a threshold value α. In the second case – "without censorship" – in case of a combination there were used all available sources.

For calculation of errors of the forecasts received by means of various aggregating methods on each security *stock*, the functionality of mean absolute error (MAE) of forecasting was used

$$\mathrm{MAE}_{stock}(Crv_{forecast}) = \frac{1}{N}\sum_{t} |Crv_{real}(stock, t) - Crv_{forecast}(stock, t)|,$$

in which forecast value of a relative share price is calculated as follows:

– consensus forecast (CF) or weighted consensus forecast (WCF, with weights that are equal to reliability of recommendations in previous periods);
– $\underline{\mathrm{E}}[F]$, $\overline{\mathrm{E}}[F]$, $\mathrm{E}_0[F] = \frac{1}{2}\left(\underline{\mathrm{E}}[F] + \overline{\mathrm{E}}[F]\right)$ values received by means of the Dempster's rule with/without discounting, with/without censorship, optimistic-neutral-pessimistic attitude to new sources.

Also, the average error for various methods of aggregating according to all papers was introduced:

$$\mathrm{MAE}(Crv_{forecast}) = \frac{1}{M}\sum_{stock} \mathrm{MAE}_{stock}(Crv_{forecast}),$$

where $M = |stock|$ is a number of shares.

5 The Results

Figures 1 and 2 represent examples of forecasting, that was based on the techniques described above, for the share price of the Transneft company (TRNFP). Figure 1 show the results of applying the Dempster's rule with the choice of the least conflicting sources. Figure 2 show the results of combining with discounting in cases of neutral attitude to new sources and "with censorship" $\alpha = 0.75$.

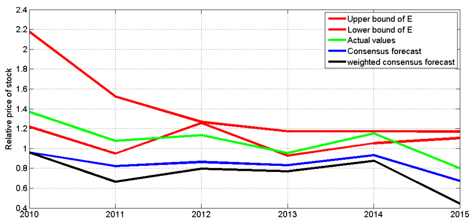

Fig. 1. The results of evidence combining by means of the Dempster rule when the least conflicting sources were chosen for the TRNFP share

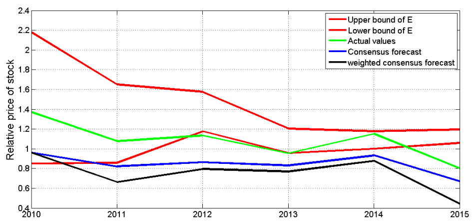

Fig. 2. The results of combining evidence with discounting, with a neutral attitude to new sources and with censorship $\alpha = 0.75$ for the TRNFP share

The mean absolute errors and of some strategies of recommendations' combining and comparison of errors to consensus forecast (CF and WCF) are presented in Table 1.

MAE_{stock}	WCF	CF	$\underline{\mathrm{E}}$, OSWC	$\overline{\mathrm{E}}$, OSWC	E_0, OSWC	$\underline{\mathrm{E}}$, NSWC	$\overline{\mathrm{E}}$, NSWC	E_0, NSWC	$\underline{\mathrm{E}}$, CLCS	$\overline{\mathrm{E}}$, CLCS	E_0, CLCS
GAZP	0,941	0,456	0,686	0,715	0,653	0,745	0,772	0,711	**0,275**	**0,359**	**0,308**
LKOH	0,499	0,27	**0,19**	0,99	0,59	**0,234**	0,918	0,524	**0,193**	0,309	**0,263**
ROSN	0,587	0,412	**0,153**	0,486	**0,302**	**0,194**	0,529	**0,265**	**0,11**	**0,257**	**0,191**
SBER	0,55	0,413	**0,137**	0,539	**0,33**	**0,178**	0,633	**0,349**	**0,116**	**0,319**	**0,233**
MAGN	0,298	**0,205**	0,385	0,937	0,661	0,347	1,014	0,681	0,345	0,47	0,422
SNGSP	0,485	0,332	**0,327**	0,84	0,546	**0,243**	1,037	0,598	**0,263**	0,405	**0,33**
GMKN	0,512	0,46	**0,382**	0,673	**0,447**	**0,349**	0,672	**0,345**	**0,283**	**0,314**	**0,267**
VTBR	0,637	0,249	0,451	0,376	0,344	0,525	0,273	0,286	0,269	**0,213**	**0,207**
TRNFP	0,33	0,234	**0,146**	0,356	**0,18**	**0,199**	0,416	**0,18**	**0,119**	**0,215**	**0,141**
TATN	0,568	0,3	**0,198**	0,998	0,589	**0,183**	1,027	0,596	**0,2**	0,421	0,331
MTSS	0,516	0,371	**0,268**	**0,352**	**0,249**	0,39	0,397	**0,259**	**0,214**	**0,205**	**0,210**
CHMF	0,311	0,203	**0,17**	0,369	0,227	**0,196**	0,405	0,222	**0,192**	0,24	0,225
ALRS	0,216	0,14	**0,119**	0,204	**0,116**	0,156	0,308	0,16	**0,115**	0,162	**0,11**
NVTK	0,236	0,395	0,28	0,396	**0,196**	0,28	0,495	**0,149**	0,273	**0,198**	**0,178**
AFLT	0,123	**0,033**	0,477	0,186	0,222	0,552	0,365	0,193	0,376	0,076	0,218
URKA	0,6	0,523	**0,504**	**0,216**	**0,357**	0,654	**0,285**	**0,339**	**0,338**	**0,244**	**0,271**
MAE	0,463	0,312	**0,305**	0,539	0,376	0,339	0,597	0,366	**0,23**	**0,275**	**0,244**

Table 1. The mean absolute errors (MAE_{stock} and MAE)

Remark 1. WCF – Weighted consensus forecast; CF – consensus forecast; the Dempster's rule with discounting and (a) optimistic scenario without censorship (OSWC); (b) neutral scenario without censorship (NSWC); (c) with the choice of the least conflicting sources (CLCS).

The consensus forecast turned out to be more accurate than all other methods for only two stocks (MAGN, AFLT). In other cases, one of the aggregating methods with using the Dempster's rule turned out to be more accurate than the consensus forecast (CF). The results of calculating of the MAE for each stock, taking into account discounting with an optimistic and neutral attitude to new sources without introducing a confidence threshold and with a threshold value, differ little from each other. For a pessimistic attitude to new sources with a threshold value of reliability and without it, the difference is more significant, the predicted value of $\underline{\mathrm{E}}[F]$ turns out to be more accurate in most cases without the introduction of censorship, and the estimate $\mathrm{E}_0[F]$ is opposite, more accurate in censoring. In most cases forecasts $\underline{\mathrm{E}}[F]$ and $\mathrm{E}_0[F]$, obtained by using the Dempster's rule without discounting with the choice of the least conflicting sources, turned out to be more accurate than similar estimates with discounting under a neutral or pessimistic attitude to new sources. With a positive attitude to new sources and discounting, the estimate $\underline{\mathrm{E}}[F]$ was more accurate than the analogous estimate obtained with the Dempster's rule without discounting. But in this case estimates $\mathrm{E}_0[F]$ and $\bar{\mathrm{E}}[F]$ were more precise with the Dempster's rule without discounting. For at least half of the shares, the most accurate estimate was $\underline{\mathrm{E}}[F]$, obtained with the help of Dempste's rule. Estimates obtained by using discounting with a positive attitude to new sources and in the absence of censorship, and estimates obtained according to Dempster's rule when choosing the least conflicting sources had the smallest average error for all considered shares.

The results of the combination obtained with discounting under different scenarios of the relation to new sources and in the absence of a threshold value of reliability and with it do not show significant differences. In this case, the intervals of the expected values of the comparative price of the stock are wider than for the Dempster's rule without discounting.

6 Summary and Conclusion

The aggregation method of financial analysts' recommendations based on the evidence theory was considered in this paper on the example of Russian stock market. The comparison of obtaining results with consensus method was performed. Various strategies of combining within the framework of the evidence theory was suggested and tested. The lower bounds or mean values of the expected value interval of the relative price obtained by using the Dempster's rule with discounting turned out to be most accurate aggregate estimates. However, relying on the obtained results, it is impossible to say unambiguously which of the combination rules is the universal and most accurate.

In terms of further research, it is of interest to aggregate not only the Dempster rule, but also other rules of the theory of evidence that more fully reflect the nature of decisions (for example, pessimism-optimism), the formation of a "portfolio" of aggregated data source based on certain optimality criteria and constraints on conflict , uncertainty, etc.

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