Recognition of Cross Profiles of Roadbed Based on Polygonal Representations

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Abstract. The paper is devoted to the description of two approaches for recognizing railway roadbed profiles, obtained with the help of laser scanning. The first approach is based on the identification of similar parts of comparing profiles, presented by their polygonal representations. The second approach uses weighted functional metrics, where it is possible to process incomplete data and then to make a choice on the set of preferences.

1 Introduction

Nowadays there are several program systems that allow to monitor automatically railway roads. These systems give possibility to find defects linked with the track structure, gabarite dimensions, rail cross profiles, etc. The full information about it can be found in [1, 2, 8, 9]. The development of such systems is produced in two directions with the help of using new hardware or new software to achieve higher monitoring speed and accuracy.

In the paper we consider the problem of roadbed profiles recognition that is then used for detecting their defects, like non-normative breadth of ballast shoulder, non-normative breadth of roadbed shoulder, places of oversized angles of slope. The review of the possible approaches of detecting such defects can be found in [1]. We assume that polygonal representations of measured cross profiles of the roadbed are the input data for our methods. These polygonal representations can be evaluated statistically [4] by using clouds of points obtained by laser devices.

2 Data description: cross profiles of roadbed

The ideal cross profile of the roadbed can be represented as a polygon. Meanwhile, a real cross profile considerably differs from an ideal one, but it is possible to approximate its geometrical form by the sequence of straight segments. As we mention before, there is a problem of finding defects of the roadbed, and it can be solved by comparing the measured profile with the normative profile according to the design decision. In Russia there are special normative laws called "Constructional Norms and Rules", see [6] for example. There are some differences in solving such a problem linked with prior information in our disposal. The ideal situation is when the design decision is known. Then it is necessary to compare the etalon profile based on the decision design with the measured one to detect the set of possible defects. If the design decision is not known we need to recognize the measured profile using the set of all possible etalon profiles. In this case we have the set of etalon profiles that correspond to different types of roadbed that can be classified as ditch cuts, embankments, tunnels, etc. Examples of such etalon profiles are shown on Fig. 1.

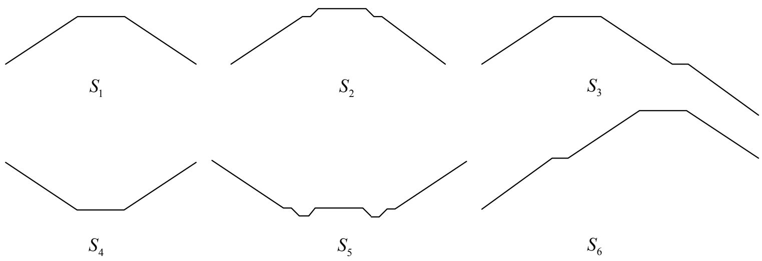

Fig. 1. Six etalon profiles

The analysis of the cross profile of the roadbed assumes that we need to extract feature points, whose positions determine the basic characteristics of the roadbed. The feature points are points corresponding to edges of upper and lower surfaces of embankment (ditch cut) of the roadbed and the ballast section. These points can be extracted by the statistical method of recovering profile described in [4]. It is worth to mention that such points are extracted with different plausibility and reliability. It was experimentally checked that points describing the ballast section are extracted with the better degree of plausibility.

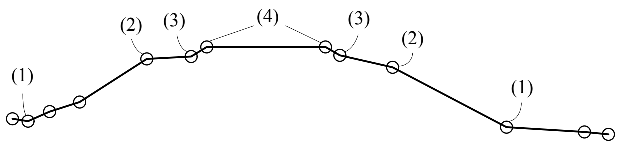

Fig. 2. The measured cross profile of the road bed with extracted feature points

The example of the measured profile is depicted on Fig. 2 with its polygonal representation. Each vertex of this representation can be interpreted as a feature point. One can see there edges of lower (1) and upper (2) surfaces, edges of lower (3) and upper (4) surfaces of the ballast section.

3 Recognition of road bed profiles based on their polygonal representations

3.1 The problem statement

Given etalon profiles $S_1, ..., S_N$ of the roadbed. Each profile S_j, $j = 1, ..., N$, is defined by a polygonal representation $S_j = \{\mathbf{s}_1^{(j)}, ..., \mathbf{s}_{n_j}^{(j)}\}$, where $\mathbf{s}_i^{(j)} = (u_i^{(j)}, v_i^{(j)})$, $i = 1, ..., n_j$, $j = 1, ..., N$, are vertices of the corresponding polygon. We assume that $u_1^{(j)} < u_2^{(j)} < ... < u_{n_j}^{(j)}$. It is necessary to recognize the measured profile that is also described by the polygonal representation $X = \{\mathbf{x}_1, ..., \mathbf{x}_m\}$, where $\mathbf{x}_i = (x_i, y_i)$, $i = 1, ..., m$, and $x_1 < x_2 < ... < x_m$.

The classification problem consists in finding etalon profile S_j that is similar to the measured profile X according to the chosen distant function. After that it is possible to compute the important characteristics of X like the height of embankment or the depth of ditch cut, and others.

We will describe below two algorithms for solution of this classification problem.

3.2 The comparison of polygonal representations based on invariants

If we compare polygonal representations X and S_k, then they can differ in number of vertices, and in choice of coordinate system. To solve this problem, we propose to normalize them, i.e. to find a coordinate system, where considered polygonal representations are the most similar. This can be produced by the approach, described in [5]. Let $X^{(p,l)} = \{\mathbf{x}_i\}_{i=p}^{p+l-1}$ be a part of X that consists of l points. We will characterize any such representation $Y = X^{(p,l)}$ by the following vectors of features that are invariant w.r.t. the choice of coordinate system:

1) $\mathbf{r}(Y) = (r_i(Y))_{i=1}^{l-2}$, where $r_i(Y) = |\mathbf{r}_{i+2} - \mathbf{r}_{i+1}|/|\mathbf{r}_{i+1} - \mathbf{r}_i|$ is the ratio of lengths of two neighboring segments;

2) $\beta(Y) = (\beta_i(Y))_{i=1}^{l-2}$, where $\beta_i(Y)$ is the angle between neighboring segments $[\mathbf{r}_i, \mathbf{r}_{i+1}]$ and $[\mathbf{r}_{i+1}, \mathbf{r}_{i+2}]$;

3) $\sigma(Y) = (\sigma_i(Y))_{i=1}^{l-2}$, where $\sigma_i(Y)$ is the sign of angle between neighboring segments $[\mathbf{r}_i, \mathbf{r}_{i+1}]$ and $[\mathbf{r}_{i+1}, \mathbf{r}_{i+2}]$.

Let $X_1^{(p_1,l_1)}$ and $X_2^{(p_2,l_2)}$ be parts of polygonal representations. Then they are called *comparable* if $l_1 = l_2$ and $\sigma\left(X_1^{(p_1,l_1)}\right) = \sigma\left(X_2^{(p_2,l_2)}\right)$. Let $G_0 = \left\{\left(X^{(p,l)}, S_k^{(m,l)}\right)\right\}$ be the set of comparable parts of polygonal representations of the measured profile and etalon profiles. Then we can compute the similarity of comparable representations using the Euclidean metric

$$\rho\left(X^{(p_1,l)}, S_k^{(p_2,l)}\right) = w_1 \sqrt{\left\|\mathbf{r}\left(X^{(p_1,l)}\right) - \mathbf{r}\left(S_k^{(p_2,l)}\right)\right\|^2 + w_2 \left\|\beta\left(X^{(p_1,l)}\right) - \beta\left(S_k^{(p_2,l)}\right)\right\|^2},$$

where $w_1 > 0$ is the coefficient allowing us to increase the preference of some pairs (for example, pairs with a large number of segments), $w_2 > 0$ is the coefficient of scaling of vectors $\mathbf{r}$ and β.

Let

$$\Upsilon_k = \left\{ \rho \left(X^{(p_1,l)}, S_k^{(p_2,l)} \right) : \left(X^{(p_1,l)}, S_k^{(p_2,l)} \right) \in G_0 \right\}, k \in \{1, ..., N\},$$

be the ordered set of distances, where $\rho \left(X^{(p_1{}^{(1)},l^{(1)})}, S_k^{(p_2{}^{(1)},l^{(1)})} \right)$ is after $\rho \left(X^{(p_1{}^{(2)},l^{(2)})}, S_k^{(p_2{}^{(2)},l^{(2)})} \right)$ if $p_1^{(1)} > p_1^{(2)}$ or $\left(p_1^{(1)} = p_1^{(2)} \right) \wedge \left(l^{(1)} \geq l^{(2)} \right)$. According to the algorithm, we should detect local minima in vectors $\Upsilon_k = \{\rho_i\}$, $k = 1, ..., N$. The set of pairs, that correspond to these minima, is denoted by G_1. For each pair $\left(X^{(p,l)}, S_k^{(m,l)} \right) \in G_1$ we normalize polygonal representations X and S_k by finding the coordinate system that corresponds to similar parts $X^{(p,l)}$ and $S_k^{(m,l)}$. This leads to the following optimization problem

$$\left\| AX^{(p,l)} - S_k^{(m,l)} - B \right\|^2 \to \min$$

w.r.t. orthogonal transformation A and vector $B \in R^2$. The analytical solution of this optimization problem can be found in [5]. After that we can compute the normalized representation of X by the formula

$$\tilde{X} = AX^{(p,l)} - B.$$

After that we should find the best normalization among possible ones. This is produced by finding a distance between the normalized profile $\tilde{X}$ and the etalon profile S_k. One of possible ways to do this is based on distance images [7]. In this case polygons $\tilde{X}$ and S_k are depicted as binary images. Let $Y = \{y_{i,j}\}_{N_1 \times N_2}$, where $y_{i,j} \in \{0, 1\}$, be a binary image. Then the distance image is the matrix $Y^* = \{y^*_{i,j}\}_{N_1 \times N_2}$ computed by

$$y^*_{i,j} = \min_{(k,m)|y_{k,m}=1} \sqrt{(i-k)^2 + (j-m)^2}.$$

Let Y_1 and Y_2 be polygons and let $Y_1{}^*$ and $Y_2{}^*$ be the corresponding distance images. Then the closure between Y_1 and Y_2 can be measured by

$$\mu(Y_1, Y_2) = \frac{\|Y_1^* - Y_2^*\|}{\max\{\|Y_1^*\|\,;\|Y_2^*\|\}},$$

where $\| \;\|$ is the matrix norm. Let us notice that there are computationally effective algorithms that approximately compute distance images [3]. Let $\tilde{X}_k$ be the best normalization of X w.r.t. etalon image S_k providing the minimum of μ. Then the classification rule can be defined as follows: the measured profile X corresponds to the etalon S_m if $\mu(\tilde{X}_m, S_m) \leq \mu(\tilde{X}_k, S_k)$, $k = 1, ..., N$.

Let us assume that the set of all etalon profiles is depicted on Fig. 1, and the measured contour X is depicted on Fig. 2. Then applying the described classification algorithm we get that the measured contour corresponds to the etalon S_2. Fig. 3 shows the similar parts of comparing polygonal representations and how the polygon $\tilde{X}_2$ is fitted to S_2.

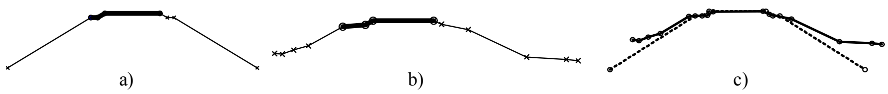

Fig. 3. Parts of polygonal representations: a) optimal $S_2^{(m,l)}$ (bold line); b) optimal $X^{(p,l)}$ (bold line); c) S_2 and $\tilde{X}_2$

3.3 The comparison of polygonal representations by norms on the curve space

Obviously, any polygon can be considered as a piecewise linear function. Therefore, there is a possibility for comparing polygons using a norm defined in a space of functions. For example, we can consider the normalized space $C_p^w[a,b]$ of continuous functions on $[a,b]$ with the norm $\|f\|_{w,p} = \left(\int_a^b w(t)|f(t)|^p dt\right)^{\frac{1}{p}}$, $1 \le p < \infty$, $\|f\|_{w,\infty} = \max\limits_{t\in[a,b]} w(t)\,|f(t)|$, where w is a non-negative weight function such that $\int_a^b w(t)dt = 1$. Let us assume that we can detect feature points $\mathbf{x}^{(0)} = (x^{(0)}, y^{(0)})$, $\mathbf{x}^{(1)} = (x^{(1)}, y^{(1)})$ of the measured profile X that correspond to edges of the upper surface of embankment or the lower surface of ditch cut. Obviously we can detect the corresponding points $\mathbf{s}^{(0)} = (u^{(0)}, v^{(0)})$ and $\mathbf{s}^{(1)} = (u^{(1)}, v^{(1)})$ in the etalon representations. Using these points we can produce the normalizations $\hat{S} = \{\hat{\mathbf{s}}_1, ..., \hat{\mathbf{s}}_n\}$ and $\hat{X} = \{\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_m\}$ of the etalon profile and the measured profile as follows:

$$\hat{u}_j = u_j - \tfrac{1}{2}(u^{(0)} + u^{(1)}), \hat{v}_j = v_j - \tfrac{1}{2}(v^{(0)} + v^{(1)}), \;\; j = 1, ..., n,$$

$$\hat{x}_i = \frac{\left\|\mathbf{s}^{(0)} - \mathbf{s}^{(1)}\right\|}{\left\|\mathbf{x}^{(0)} - \mathbf{x}^{(1)}\right\|}\left(x_i - \tfrac{1}{2}(x^{(0)} + x^{(1)})\right), \hat{y}_i = \frac{\left\|\mathbf{s}^{(0)} - \mathbf{s}^{(1)}\right\|}{\left\|\mathbf{x}^{(0)} - \mathbf{x}^{(1)}\right\|}\left(y_i - \tfrac{1}{2}(y^{(0)} + y^{(1)})\right),$$

$i = 1, ..., m$, where $\hat{\mathbf{s}}_j = (\hat{u}_j, \hat{v}_j)$, $j = 1, ..., n$; $\hat{\mathbf{x}}_i = (\hat{x}_i, \hat{y}_i)$, $i = 1, ..., m$.

For notation simplicity, let us assume that polygonal representations X and S are normalized. Then the distance between X and S can be computed by the following algorithm:

1) compute the union of sequences $\{x_i\}_{i=1}^m$ and $\{u_j\}_{j=1}^n$ such that $Z = \{z_k\}_{k=1}^l = \{x_i\}_{i=1}^m \cup \{u_j\}_{j=1}^n$: $z_1 < z_2 < ... < z_l$;

2) since any polygon is a piecewise linear function, the values of such functions are computed for profiles X and S for any point $z_k \in Z$. As result we get a profiles $X = \{\mathbf{x}_1, ..., \mathbf{x}_l\}$ and $S = \{\mathbf{s}_1, ..., \mathbf{s}_l\}$ correspondently, where $\mathbf{x}_i = (z_i, \tilde{y}_i)$, $\mathbf{s}_i = (z_i, \tilde{v}_i)$.

3) the transformations made in 1) and 2) allow us to compute the distances using formulas

$$d_{p,w}(X,S) = \sqrt[p]{\sum\nolimits_{i=1}^{l-1} w_i |\tilde{y}_i - \tilde{v}_i|^p (z_{i+1} - z_i)}, \;\; 1 \leq p < \infty,$$

$$d_{\infty,w}(X,S) = \max_{1 \leq i \leq l} w_i \left| \tilde{y}_i - \tilde{v}_i \right|,$$

where weights $w_i > 0$ obeying $\sum_{i=1}^{l} w_i = 1$. Let us notice that the choice of w_i can decrease the influence of points with a small informativity that are located far from the track. If $w_i = 1$ for any i, then the corresponding metric is denoted d_p. Let us notice that the metric d_1 is proportional to quantity of work which must be made to correct of roadbed.

Let we apply metrics $d_1(X,S_k)$ and $d_{1,w}(X,S_k)$ with $w_i = w_0 \left(1 + \left|\frac{1}{2}l - i\right|^4\right)^{-1}$, $k = 1, ..., 6$, to the measured profile X on Fig. 2, and etalon profiles on Fig. 1. In this case the corresponding distances are given in Table 1.

Table 1. Values of distances $d_1(X,S_k)$ and $d_{1,w}(X,S_k)$ $k = 1, ..., 6$

	S_1	S_2	S_3	S_4	S_5	S_6
$d_1(X,S_k)$	41,5	28,9	78,8	105,9	77,5	157
$d_{1,w}(X,S_k)$	1,28	0,38	0,59	4,63	1,28	1,18

Thus, the etalon profile S_2 by both metrics has better fitted for X.

Next, we should select the most preferred profile. We say that the etalon profile S_i is more preferable than the profile S_j for the classification of the measured profile X if $d(X,S_i) < d(X,S_j)$. This fact is denoted by $S_i \succ S_j$. If we have $d(X,S_i) = d(X,S_j)$ then we say that these profiles have the same preferences and denote it by $S_i \simeq S_j$. It is possible to use the threshold comparison to enhance the robustness of the comparison procedure: $S_i \succ S_j$ if $d(X,S_i) \leq d(X,S_j) - \varepsilon$ and $S_i \simeq S_j$ if $|d(X,S_i) - d(X,S_j)| < \varepsilon$, where $\varepsilon > 0$ is a threshold value.

Then we have the following ordering of etalon profiles (and related classes) by preference in according to the metric d_1: $S_2 \succ S_1 \succ S_5 \succ S_3 \succ S_4 \succ S_6$. We have the following ordering at 5% threshold comparison (i.e. ε is equal 5% from the $\max_k d_1(X,S_k) - \min_k d_1(X,S_k)$): $S_2 \succ S_1 \succ S_5 \simeq S_3 \succ S_4 \succ S_6$. The ordering of etalon profiles is different for the specified weighted metrics $d_{1,w}$: $S_2 \succ S_3 \succ S_6 \succ S_1 \simeq S_5 \succ S_4$ for the nonthreshold comparison and $S_2 \simeq S_3 \succ S_6 \simeq S_1 \simeq S_5 \succ S_4$ for 5% threshold comparison.

4 Summary and Conclusion

In the paper we present two algorithms for recognizing the cross profile of the roadbed. Clearly both algorithms have the similar procedures: 1) normalization of polygonal representations; 2) the recognition of the measured profile based on distance function. The choice of the algorithm can be done taking into account the prior information (available or not information about feature points or other features that allows us to apply the simple algorithm for normalization) and stability of considered metrics to the noise.

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